

# CLASSIFIED PUBLICATIONS ON COVERING SYSTEMS

COLLECTED BY ZHI-WEI SUN

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## 0. SURVEYS OF PROBLEMS AND RESULTS

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## 1. ON DISJOINT SYSTEMS AND DISJOINT COVERING SYSTEMS

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7. ON COVERS BY BEATTY SEQUENCES AND INFINITE COVERS OF  $\mathbb{Z}$ 

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## 8. APPLICATIONS OF COVERS OF $\mathbb{Z}$

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