

求解问题:

$$\begin{aligned} -\nabla \cdot (a(u)\nabla u) &= 1 && \text{在}\Omega\text{内}, \\ u &= 0 && \text{在}\partial\Omega, \end{aligned}$$

其中 $\Omega = (0, 1) \times (0, 1)$. $a(s)$ 由下面定义:

$$a(s) = \int_{\Omega} s \cdot w(s, y_1, y_2) dy_1 dy_2,$$

$w(s, y_1, y_2)$ 满足:

$$\begin{aligned} -\frac{\partial}{\partial y_1} \left((s^2 + 1) \frac{\partial w}{\partial y_1} \right) - \frac{\partial}{\partial y_2} \left((s^6 + 2) \frac{\partial w}{\partial y_2} \right) &= 1 && \text{在}\Omega\text{内}, \\ w &= 0 && \text{在}\partial\Omega. \end{aligned}$$