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EXPLORE THE FASCINATING AREA OF ALGEBRAIC K-THEORY!

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# Representation Theory and Higher Algebraic K-Theory

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## The definitive resource for algebraic K-theory

**Representation Theory and Higher Algebraic K-Theory** is the first book to present higher algebraic  $K$ -theory of orders and group rings as well as characterize higher algebraic  $K$ -theory as Mackey functors that lead to equivariant higher algebraic  $K$ -theory and their relative generalizations. Thus, this book makes computations of higher  $K$ -theory of group rings more accessible and provides novel techniques for the computations of higher  $K$ -theory of finite and some infinite groups.

Authored by a premier authority in the field, the book begins with a careful review of classical  $K$ -theory, including clear definitions, examples, and important classical results. Emphasizing the practical value of the usually abstract topological constructions, the author systematically discusses higher algebraic  $K$ -theory of exact, symmetric monoidal, and Waldhausen categories with applications to orders and group rings and proves numerous results. He also defines profinite higher  $K$ - and  $G$ -theory of exact categories, orders, and group rings. Providing new insights into classical results and opening avenues for further applications, the book then uses representation-theoretic techniques—especially induction theory—to examine equivariant higher algebraic  $K$ -theory, their relative generalizations, and equivariant homology theories for discrete group actions. The final chapter unifies Farrell and Baum-Connes isomorphism conjectures through Davis-Lück assembly maps.

## Features

- Presents higher algebraic  $K$ -theory of orders and group rings for the first time in book form
- Explores connections between  $C_G$  and higher algebraic  $K$ -theory of  $C$  for suitable categories, such as exact, symmetric monoidal, and Waldhausen
- Collects methods that have been known to work for computations of higher  $K$ -theory of noncommutative rings, such as orders and group rings
- Describes all higher algebraic  $K$ -theory as Mackey functors that lead to equivariant higher algebraic  $K$ -theory and their relative generalizations for finite, profinite, and compact Lie group actions
- Obtains results on higher  $K$ -theory of orders  $\Lambda$ , and hence group rings, for all  $n \geq 0$
- Uses computations of higher  $K$ -theory of orders that automatically yield results on higher  $K$ -theory of  $RG$  ( $G$  finite) to produce results on higher  $K$ -theory of some infinite groups
- Provides appendices with many known computations and open problems in classical and higher algebraic  $K$ -theory of orders, group rings, and related structures

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