

A volume in the *Pure and Applied Mathematics* series Series edited by

Features

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# The definitive resource for algebraic *K*-theory

**Representation Theory and Higher Algebraic K-Theory** is the first book to present higher algebraic *K*-theory of orders and group rings as well as characterize higher algebraic *K*-theory as Mackey functors that lead to equivariant higher algebraic *K*-theory and their relative generalizations. Thus, this book makes computations of higher K-theory of group rings more accessible and provides novel techniques for the computations of higher *K*-theory of finite and some infinite groups.

Authored by a premier authority in the field, the book begins with a careful review of classical *K*-theory, including clear definitions, examples, and important classical results. Emphasizing the practical value of the usually abstract topological constructions, the author systematically discusses higher algebraic *K*-theory of exact, symmetric monoidal, and Waldhausen categories with applications to orders and group rings and proves numerous results. He also defines profinite higher *K*- and *G*-theory of exact categories, orders, and group rings. Providing new insights into classical results and opening avenues for further applications, the book then uses representation-theoretic techniques—especially induction theory—to examine equivariant higher algebraic *K*-theory, their relative generalizations, and equivariant homology theories for discrete group actions. The final chapter unifies Farrell and Baum-Connes isomorphism conjectures through Davis-Lück assembly maps.

# • Presents higher algebraic *K*-theory of orders and group rings for the first time in book form

- Explores connections between  $C_{G}$  and higher algebraic *K*-theory of *C* for suitable categories, such as exact, symmetric monoidal, and Waldhausen
- Collects methods that have been known to work for computations of higher *K*-theory of noncommutative rings, such as orders and group rings
- Describes all higher algebraic *K*-theory as Mackey functors that lead to equivariant higher algebraic *K*-theory and their relative generalizations for finite, profinite, and compact Lie group actions
- Obtains results on higher *K*-theory of orders  $\Lambda$ , and hence group rings, for all  $n \ge 0$
- Uses computations of higher *K*-theory of orders that automatically yield results on higher *K*-theory of *RG*(*G* finite) to produce results on higher *K*-theory of some infinite groups
- Provides appendices with many known computations and open problems in classical and higher algebraic *K*-theory of orders, group rings, and related structures

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- Some fundamental results on  $K_0$  of exact and Abelian categories

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# ontents

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