

This learning seminar aims to bring together senior undergraduates and graduate students who are interested in algebraic geometry to study foundations of vector bundles.

1. BACKGROUND

Vector bundles are widely studied geometric objects in algebraic geometry, differential geometry, representation theory and number theory. Algebraically, vector bundles can be viewed as globalization of finitely generated projective modules over commutative rings. For example they form an exact category like projective modules. Vector bundles are examples of coherent sheaves with very nice property, i.e. dualizable.

Given a projective variety X and a vector bundle V on X , there are two invariants associate to V , the sheaf cohomology $H^\bullet(X, V)$ and the characteristic class $\text{ch}(V)$. The sheaf cohomology is an algebraic/analytic invariant while the characteristic class is of topological nature. They are related by the Riemann-Roch formula. Vector bundles interact with its underlying space in two ways. There are some *natural* vector bundles that capture geometric and topological information of the underlying space X . A typical example is the bundle of algebraic differential forms Ω_X^i (when X is smooth). Its cohomology group admits a rich structure and is an important invariant of X . This is studied in *Hodge theory*. The second approach is to study the *moduli space* of vector bundles with an appropriate condition. It turns out that many properties of X can be deduced from properties of the moduli space of vector bundles on X . We will take the second approach in this seminar.

2. PLAN

I want to split the topics into three parts.

- **General theory** hilbert polynomial, (semi-)stability, Harder-Narasimahan filtration, S-equivalences and so on.
- **Moduli** flat family, infinitesimal deformation and construction of moduli stack.
- **Examples** vector bundles over smooth projective curves and their moduli spaces.

We are expected to have around ten participants, meeting once a week beginning on mid Feb 2022. In a meeting, we will have two presentations with a break between. I will distribute to every participant one topic from each part, i.e. *general theory*, *moduli* and *examples*. Everyone needs to give three presentations during the semester.

3. PREREQUISITES AND REFERENCES

I welcome audiences of all levels. For those who want to make presentations, I expect you know basic language of algebraic geometry and commutative algebra, including *ring spectrum*, *scheme* and *coherent sheaf*, i.e. chapter one and two of *Hartshorne* or equivalent. Our main reference is the book of Huybrecht and Lehn [1]. This book has free (legal) access on the internet. For the moduli stack of vector bundles on curves, I refer to a survey article of Heinloth [2], which is also available online.

REFERENCES

- [1] Huybrechts, Daniel, and Manfred Lehn. *The geometry of moduli spaces of sheaves*. Cambridge University Press, 2010.
- [2] Heinloth, Jochen. *Lectures on the moduli stack of vector bundles on a curve*. In *Affine flag manifolds and principal bundles*, pp. 123–153. Springer, Basel, 2010.