8.1.4 ($\varepsilon=-1$, 解决 (b) (i))

Here $\mathcal{B}(V_1) \simeq \Lambda(V_1)$ is an exterior algebra.

Consequently $\text{ш}_{m,n}$, $m,n \in \{0,1\}$ generates K^1.

It's easy to check that

(8.14) $g_2 \cdot z_1 = q_{21} q_{22} (z_1 + w_1)$

$\partial_3(z_1) = (1-\tilde{q}_{12}) x_2 - \tilde{q}_{12} x_1$

(8.15) $\delta(z_1) = g_1 g_2 \otimes z_1 + ((1-\tilde{q}_{12}) x_2 - \tilde{q}_{12} x_1) g_2 \otimes x_3$

8.14: $g_2 \cdot z_1 = g_2 \cdot (x_2 x_3 - q_{12} x_3 x_2)$

$= q_{21}(x_2 + x_1) q_{22} x_3 - q_{12} q_{22} x_3 q_{21} (x_2 + x_1)$

$= q_{21} q_{22} (x_2 x_3 + x_1 x_3 - q_{12} x_3 x_2 - q_{12} x_3 x_1)$

$= q_{21} q_{22} (z_1 + w_1)$

8.15: $\delta(z_1) = \delta(x_2 x_3 - q_{12} x_3 x_2)$

$\Delta(x_2) = x_2 \otimes 1 + g_1 \otimes x_2$

$\delta(x_3) = x_3 \otimes 1 + g_2 \otimes x_3$

~~Then~~

Case 1: $\tilde{q}_{12} = 1$

$w_1 = 0$ by (8.7) (8.8)

hence $\text{ш}_{1,1} = -(\mathrm{ad}_c x_2) w_1 = 0$

In consequence, $z_0 = x_3$ and z_1 form a basis of K^1.

Suppose $k_1 x_3 + k_2 z_1 \in \mathcal{B}(V)$

$\partial_1 \partial_3 (k_1 x_3 + k_2 z_1) = \cdots = k_{k+1}$

Thus $c(x_3 \otimes x_3) = q_{22} x_3 \otimes x_3$

$c(x_3 \otimes z_1) = q_{21} q_{22} z_1 \otimes x_3$

$c(z_1 \otimes x_3) = q_{12} q_{22} x_3 \otimes z_1$

$c(z_1 \otimes z_1) = -q_{22} z_1 \otimes z_1$

$c(x_3 \otimes z_1) = g_2 \cdot z_1 \otimes x_3$

$= q_{21} q_{22} (z_1 + w_1) \otimes x_3$, $w_1 = 0$

$c(z_1 \otimes x_3) = g_1 g_2 \cdot x_3 \otimes z_1 - (x_1 g_2) \cdot x_3 \otimes x_3$

$g_1 g_2 \cdot x_3 = q_{12} q_{22} x_3$

$(x_1 g_2) \cdot x_3 = \mathrm{ad}_c x_1 (q_{22} x_3)$

$= q_{22} w_1 = 0.$

$= q_{12} q_{22} x_3 \otimes z_1$

Thus K^1 is of diagonal type

q_{22} —— q_{22}^2 —— $-q_{22}$

If $q_{22} \neq \pm 1$, this diagram doesn't appear in Table 1 [H2]

and we conclude by our hypothesis 1.7

$\mathrm{GKdim}\, K = \infty$

$\Rightarrow \mathrm{GKdim}\, \mathcal{B} = \infty$

For $q \in \Bbbk^\times$, let $\mathfrak{C}_\pm(q) = V$

be the braided v.s as in (8.3)

under the assumption that $\varepsilon = -1$

$q_{12} = q = q_{21}^{-1}$

$q_{22} = \pm 1$

We call $\mathcal{B}(\mathfrak{C}_\pm(q))$ and $\mathcal{B}(\mathfrak{C}_\pm(q))$

the Endymion algebras.

Prop 8.6 (i.e. $q_{22} = 1$)

The algebra $\mathcal{B}(\mathfrak{C}_+(q))$ is presented by generators x_1, x_2, x_3 and relations

(8.17) $x_1^2 = 0$, $x_2^2 = 0$, $x_1 x_2 = -x_2 x_1$

(8.18) $(x_2 x_3 - q x_2 x_3)^2 = 0$

(8.19) $x_3 (x_2 x_3 - q x_3 x_2) = q^{-1} (x_2 x_3 - q x_3 x_2) x_3$

(8.20) $x_1 x_3 = q x_3 x_1$

Let $z_1 = x_2 x_3 - q x_3 x_2$

Then $\mathcal{B}(\mathfrak{C}_+(q))$ has a PBW-basis

$B = \{ x_1^{m_1} x_2^{m_2} x_3^{m_3} z_1^{m_4} : m_1, m_2, m_4 \in \{0,1\}, m_3 \in \mathbb{N}_0 \}$

Hence $\mathrm{GK} = 1$.

Proof: (8.17) because $\mathcal{B}(V_1) \cong \Lambda(V_1)$

(8.18) i.e. $z_1^2 = 0$

By (8.16) $c(z_1 \otimes z_1) = -z_1 \otimes z_1$

$\Rightarrow z_1^2 = 0$

(8.19) i.e. $x_3 z_1 \overset{?}{=} q^{-1} z_1 x_3$

by (8.16), $q_{22} = 1$

(8.20) $w_1 = 0$. (8.7), (8.8)

Hence the quotient $\tilde{\mathcal{B}}$ of $T(\mathfrak{C}_+(q))$ by (8.17) ~ (8.20)

projects onto $\mathcal{B}(\mathfrak{C}_+(q))$

Also the following relations hold in $\tilde{\mathcal{B}}$:

$x_1 z_1 = -q z_1 x_1$

$x_2 z_1 = -q z_1 x_2$

$x_1 z_1$: $\delta(x_1) = x_1 \otimes 1 + g_1 \otimes x_1$

$x_1 z_1 = (x_1 \cdot z_1) 1 + (g_1 \cdot z_1) x_1$

$= \mathrm{ad}_c x_1 (z_1) + g_1 (x_2 x_3 - q x_3 x_2) x_1$

$\mathrm{ad}_c x_1(z_1) = \text{ш}_{1,1} = 0$

$= -q z_1 x_1$

$g_1 \cdot x_2$: -1

$g_1 \cdot x_3$: q

Now the subspace I spanned by B is a right ideal of $\tilde{\mathcal{B}}$. $\Rightarrow I = \tilde{\mathcal{B}}$

It's remains to show B is linearly indep. in $\mathcal{B}(\mathfrak{C}_+(q))$

$\mathcal{B}(V) \cong \mathcal{B}(K^1) \# \mathcal{B}(V_1)$

As v.s $\mathcal{B}(V) \cong \mathcal{B}(K^1) \otimes \mathcal{B}(V_1)$

$\mathcal{B}(V_1) \cong \Lambda(V_1)$: $x_1^{m_1}, x_2^{m_2}$, $m_1, m_2 \in \{0,1\}$

1 $\quad 1$ $\quad -1$

○ —— ○

x_3 $\quad z_1$

0 ⊗ 0.1.

Prop 8.7: ...