
If $(G, \psi) : A \to B$ is a braided monoidal functor,

then (G, ψ) is also braided monoidal with respect to the inverse braiding of A and B

$\underline{(G, \psi)} : \bar{A} \to \bar{B}$

It follows from Prop 3.3.4 and $\underline{\text{Lemma } 12.2.9\ (1)}$

that the functors

${}_A\mathcal{C}_{rat} \to {}_{rat}\mathcal{C}_A \quad (V, \lambda) \mapsto \underline{(V, \lambda_+)}$

${}_{rat}\mathcal{C}_A \to {}_A\mathcal{C}_{rat} \quad (V, \lambda) \mapsto (V, \lambda_-)$

are well-defined inverse iso morphisms

Prop 3.3.4

${}_A\mathcal{C}_{rat} \to {}_{rat}\mathcal{C}_A$

$(V, \lambda) \mapsto (V, \lambda_+) \in {}_{rat}$?

Lemma 12.2.9 (1):

$X \in {}_R\mathcal{C}$, $(F_n X)_{n \geq 0}$ is an $\mathbb{N}_0$-filtration in $\mathcal{C}$

$\underbrace{\lambda_+} = \lambda \underbrace{c_{V,A}} (\mathrm{id}_V \otimes S_A)$

Suppose $V \in (V, \lambda) \in {}_A\mathcal{C}_{rat}$

then $V \in F_i X$ for some i

$\underline{\Rightarrow V_0 \in F_i(X)}$

$\underline{S_A}$, $\underline{h \cdot A}$ are graded

Hence the braided monoidal isomorphisms in $\begin{pmatrix} \text{Thm } 3.4.15 \\ 3.4.16 \\ \text{Cor } 3.4.17 \end{pmatrix}$ for A

restrict to braided monoidal iso again, denoted by : ---

知识储备：

① Let $\mathcal{C}$, $\mathcal{C}'$ be braided strict monoidal cat. $(F, \varphi) : \mathcal{C} \to \mathcal{C}'$ be a braided monoidal functor

Let H be a Hopf algebra in $\mathcal{C}$,

H' the Hopf algebra $\underline{(F, \varphi)(H)}$ in $\mathcal{C}'$.

Then (F, φ) induces a braided monoidal functor $YD(F, \varphi)$

$F : YD(\mathcal{C})^H_H \to YD(\mathcal{C}')^{H'}_{H'}$

$\underline{(V, \lambda, \delta) \mapsto (F(V), \lambda', \delta')}$

$f \mapsto F(f)$

monoidal φ

② Let H, H' be Hopf algebras in $\mathcal{C}$,

S_H, $S_{H'}$ are iso,

Let $\rho : H \to H'$ be a Hopf algebra iso,

Then the functor

$YD(\rho) : YD(\mathcal{C})^H_H \to YD(\mathcal{C})^{H'}_{H'}$

$(V, \lambda, \delta) \mapsto (V, \lambda', \delta')$

$\lambda' = \lambda(\mathrm{id}_V \otimes \rho^{-1})$

$\delta' = \underline{(\mathrm{id}_V \otimes \rho) \delta}$

$f \mapsto f$

is a braided strict monoidal iso.

③ Let H be a Hopf algebra in $\mathcal{C}$, S - iso

Then H is a Hopf algebra in $\underline{\mathcal{C}^{rev}}$ C^v

Then the functors are strict monoidal iso:

$(C^{rev})_H \to ({}_H C)^{rev} \quad (V, \lambda) \mapsto (V, \lambda)$

$(C^{rev})^H \to ({}^H C)^{rev} \quad (V, \delta) \mapsto (V, \delta)$

and that

$YD(\mathcal{C}^{rev})^H_H \to ({}^H_H YD(\mathcal{C}))^{rev}$

$(V, \lambda, \delta) \mapsto (V, \lambda\delta)$

$f \mapsto f$

④ $\underline{F^{rev}_{\mathcal{C}}} : (\underline{\mathrm{id}}, \underline{c}) : \underline{\mathcal{C}^{rev} \to \mathcal{C}}$

$\underline{\bar{F}^{rev}_{\mathcal{C}}} : (\mathrm{id}, \varphi) : \mathcal{C} \to \mathcal{C}^{rev}$

$\varphi_{X,Y} = \bar{c}_{Y,X}$

$\underline{F^{rev}_{\mathcal{C}, \bar{c}}} : (\mathrm{id}, \bar{c}) : \underline{\mathcal{C}^{rev} \to \mathcal{C}}$

$\underline{\bar{F}^{rev}_{\mathcal{C}, \bar{c}}} : (\mathrm{id}, \varphi) : C \to C^{rev}$

$\varphi_{X,Y} = c_{Y,X}$

are inverse braidal monoidal iso.

$\overline{F}^{rev}_{\mathcal{C}}(H) = (H^{op})^{cop}$

$\overline{F}^{rev}_{\mathcal{C},c}(H) = (H^{cop})^{op}$

Thm3.4.15:

$(F^{YD}_{rl}, \rho): \mathcal{YD}(\mathcal{C})^H_H \to {}^H_H\mathcal{YD}(\mathcal{C})$

$(V,\lambda,\delta) \mapsto (V,\lambda_-,\delta_+)$

$\rho_{X,Y} = c^{\mathcal{YD}(\mathcal{C})^H_H}_{Y,X}\,\bar{c}_{X,Y}$

$(F^{YD}_{lr}, \psi): {}^H_H\mathcal{YD}(\mathcal{C}) \to \mathcal{YD}(\mathcal{C})^H_H$

$(V,\lambda,\delta) \mapsto (V,\lambda_+,\delta_-)$, ψ...

思路:

$S: (H^{op})^{cop} \to H$

$\mathcal{YD}(\mathcal{C})^H_H \xrightarrow{\mathcal{YD}(\overline{F}^{rev}_{\mathcal{C}})} \mathcal{YD}(\mathcal{C}^{rev})^{(H^{op})^{cop}}_{(H^{op})^{cop}} \xrightarrow{\mathcal{YD}(S)} \mathcal{YD}(\mathcal{C}^{rev})^H_H \xrightarrow{(3.4.10)} \left({}^H_H\mathcal{YD}(\mathcal{C})\right)^{rev} \xrightarrow{E^{rev}_{{}^H_H\mathcal{YD}(\mathcal{C})}} {}^H_H\mathcal{YD}$

Thm3.4.16

$\overline{F}^{YD}_{rl}: \overline{\mathcal{YD}(\mathcal{C})^H_H} \to {}^{H^{cop}}_{H^{cop}}\mathcal{YD}(\overline{\mathcal{C}})$

$(V,\lambda,\delta) \mapsto (V,\lambda_-,\bar{c}_{V,H}\delta)$

$\overline{F}^{YD}_{lr}: {}^{H^{cop}}_{H^{cop}}\mathcal{YD}(\overline{\mathcal{C}}) \to \overline{\mathcal{YD}(\mathcal{C})^H_H}$

$(V,\lambda,\delta) \mapsto (V,\lambda_+,c_{H,V}\delta)$

are braid monoidal iso.
Strict

Cor 3.4.17

$F: \overline{{}^H_H\mathcal{YD}(\mathcal{C})} \to {}^{H^{cop}}_{H^{cop}}\mathcal{YD}(\overline{\mathcal{C}})$

$(V,\lambda,\delta) \mapsto (V,\lambda,(S_H^{-1}\otimes id)\,\bar{c}^{2}_{H,V}\,\delta)$

$f \mapsto f$

$\varphi_{X,Y} = \bar{c}^{{}^H_H\mathcal{YD}(\mathcal{C})}_{Y,X}\,c_{X,Y}$

is a braided monoidal iso.

Proof:

$\overline{{}^H_H\mathcal{YD}(\mathcal{C})} \xrightarrow{(\overline{F}^{YD}_{lr},\varphi)} \overline{\mathcal{YD}(\mathcal{C})^H_H} \xrightarrow{\overline{F}^{YD}_{rl}} {}^{H^{cop}}_{H^{cop}}\mathcal{YD}(\overline{\mathcal{C}})$

Thm12.3.2 (Ω, ω)

The functor $\Omega: {}^B_B\mathcal{YD}(\mathcal{C})_{rat} \to {}^A_A\mathcal{YD}(\mathcal{C})_{rat}$

$(V,\lambda,\delta) \mapsto (V,\lambda_1,\delta_1)$

$\lambda_1 = (A\otimes V \xrightarrow{id\otimes\delta} A\otimes B\otimes V \xrightarrow{\langle,\rangle} V)$

$\delta_1 = (V \xrightarrow{\delta_2} A\otimes V \xrightarrow{S_A^2\otimes id} A\otimes V \xrightarrow{c^2_{A,V}} A\otimes V)$

where δ_2 is defined by

$\lambda = (B\otimes V \xrightarrow{id\otimes\delta_2} B\otimes A\otimes V \xrightarrow{\langle,\rangle^{+}\otimes id} V)$

$f \mapsto f$

$\omega_{X,Y} = c^{{}^B_B\mathcal{YD}(\mathcal{C})}_{Y,X}\,\overline{\sigma}_{X,Y}$

for all $X, Y \in {}^B_B\mathcal{YD}(\mathcal{C})_{rat}$,

$$\begin{tikzcd}
\overline{{}^B_B\mathcal{YD}(\mathcal{C})_{rat}} \arrow[rr, "{(\Omega,\omega)}"] \arrow[dr, "D"'] & \circlearrowleft & \overline{{}^A_A\mathcal{YD}(\mathcal{C})_{rat}} \arrow[dl, "{(\overline{F},\varphi)}"] \\
 & {}^{A^{cop}}_{A^{cop}}\mathcal{YD}(\overline{\mathcal{C}})_{rat} &
\end{tikzcd}$$

Proof:

By cor 3.4.17 $(\overline{F},\varphi)$ is an iso the inverse $(\overline{F}^{YD}_{rl}, \rho)\,\underline{\overline{F}^{YD}_{lv}}$

define (Ω, ω):

$$\begin{tikzcd}
\overline{{}^B_B\mathcal{YD}(\mathcal{C})_{rat}} \arrow[r, "D"] & {}^{A^{cop}}_{A^{cop}}\mathcal{YD}(\overline{\mathcal{C}})_{rat} \arrow[r, "\overline{F}^{YD}_{lv}"] & \overline{{}_{rat}\mathcal{YD}(\mathcal{C})^A_A} \arrow[r, leftrightarrow, "{(\overline{F}^{YD}_{rl}, \rho)}"] & \overline{{}^A_A\mathcal{YD}(\mathcal{C})_{rat}}
\end{tikzcd}$$

Let $(V, \lambda, \delta) \in {}^B_B\mathcal{YD}(\mathcal{C})_{rat}$

$$\Omega(V,\lambda,\delta) = \underline{F^{YD}_{rl}}\;\underline{\overline{F}^{YD}_{lv}}\;\underline{D}(V,\lambda,\delta)$$

$$= \underbrace{\left(V, \bar{\lambda}_{+-}, (S_A \otimes \mathrm{id}_V)\, c^{2}_{A,V}\,\delta\right)}$$

$\bar{\lambda} = (A \otimes V \xrightarrow{\mathrm{id}\otimes\delta} A \otimes B \otimes V \xrightarrow{\langle,\rangle\otimes \mathrm{id}} V)$

$\lambda = (B \otimes V \xrightarrow{\mathrm{id}\otimes\bar{\delta}} B \otimes A \otimes V \xrightarrow{\langle,\rangle^{+cop}\otimes \mathrm{id}} V)$

$\bar{\lambda}_{+-} = \bar{\lambda} = \lambda,$

We have to prove that

$$(S_A \otimes id_V)\, c^2_{A,V}\, \bar{\delta} = \delta,$$

$$(S_A \otimes id_V)\, c^2_{A,V}\, \bar{\delta} = \underline{\cancel{c^2_{A,V}}\,(S_A^2 \cancel{\otimes id})\,\delta_2}$$

that is

$$\boxed{\delta_2} = (S_A^{-2} \otimes id_V)\, \bar{c}^2_{A,V}\, (S_A \otimes id_V)\, c^2_{A,V}\, \bar{\delta}$$

$$= \boxed{(S_A^{-1} \otimes id_V)\, \bar{\delta}}$$

check :

$$X = B \otimes V \xrightarrow{id \otimes \delta_2} B \otimes V \otimes A \otimes V \xrightarrow{\langle , \rangle \otimes id} V$$

routine

Ω ✓

Let $G = \bar{F}^{YD}_{ev} D$, then

$$\Omega = \boxed{P^{YD}_{rl}}\, G$$

$$\underline{\omega_{X,Y}} = P_{G(X),G(Y)}$$

$$\| $$

$$\underline{c^{YD(\mathcal{C})^A_A}_{G(Y),G(X)}\, \bar{c}_{G(X),G(Y)}}$$

$$\| \quad \| \quad \|$$

$$c^{{}^B_BYD(\mathcal{C})}_{Y,X}\, \bar{c}_{X,Y}$$

G strict braided monoidal functor

G 不改变对换

□