大数据智能云与保险科技

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江苏省保监会讲座
2017年6月12日

江苏省保险学会文件

苏保学〔2017〕28 号

关于举办大数据智能云与保险科技专题讲座的通知

各保险公司省级分公司：

当前科技迅猛发展，保险科技（InsurTech）发展日新月异，技术给保险业带来了很大的发展空间和应用，也构成了极大的挑战。在此背景下，特邀专家学者作专场讲座。讲座由江苏保监局指导，省保险学会主办。现将相关事项通知如下：

一、时间地点

时间：2017 年 6 月 12 日（周一）下午 14:30-17:00

地点：南京市长江路 69 号人保大厦 5 楼会议室

二、参会对象

各保险公司省级分公司总经理 1 人。

邀请江苏保监局、省金融学会、各保险学会、各保险行业协会、省保险中介行业协会、各中介公司，紫金财产保险股份有限公司，乐爱金财产保险（中国）有限公司，利安人寿保险股份有限公司，东吴人寿保险股份有限公司，国联人寿保险股份有限公司派员参加。

三、主讲人员

戴万阳，南京大学数学系教授、博导，为江苏金融科技研究中心特邀专家，同时兼任国家与江苏省多个概率、统计、运筹学及工业与应用数学学会的常务理事与副理事长；国家自然科学奖励委员会数学学科会评委员、国家自然科学基金重大/重点项目评审专家。

江苏省保险学会

2017 年 6 月 6 日

抄送：江苏保监局，省金融学会，各保险学会，各保险行业协会，省保险中介行业协会，各中介公司，紫金财产保险股份有限公司，乐爱金财产保险（中国）有限公司，利安人寿保险股份有限公司，东吴人寿保险股份有限公司，国联人寿保险股份有限公司。

江苏省保险学会综合部　　2017 年 6 月 6 日印发

保险业近况

- 中国保险业已经历六十多年发展历程
  - **2016**年保费收入飙升至全球第二
  - 已成为名副其实的保险大国

- 中国保险业迎来黄金机遇期
  - 一个发达市场经济必有发达的保险市场作后盾
  - 保险业不仅是社会发展的稳定器
    - 也是经济发展的助推器。

保险科技的定义

- 它是不同保险行业的生态主体
  - 即包括传统保险公司
  - 也包括科技相关初创企业等

- 运用不同的科技产品、新的科技技术，包括
  - 大数据、区块链
  - 云计算、人工智能
  - 物联网、基因测序

- 应用“互联网+”到整个保险行业本
  - 改变行业原有痛点
  - 改进整个行业的生态。

保险科技的演化与发展

- 从金融科技演化而来
  - 初期仅是金融科技框架里的一小部分
- 2008年金融危机后
  - 很多创新成果已融化在当今的保险科技里
- 近年，保险科技融资整体不断上升
  - 尤其是2011年后，融资额度增长比例大幅上升

保险科技对保险生态的影响

- 首先，众多专门保险科技企业的相继成立
  - 传统保险公司已经意识到了保险科技或者是科技可能给他们带来非常重要的挑战
    - 众安科技、平安科技、太平电商、太平洋在线，泰康在线、中国人寿等由传统保险公司自己创立的科技公司
  - 在国外，保险公司在开设保险科技公司的同时，会专门设立一个创新事业部应对创新科技的发展
    - 美国很多融资非常成功的InsurTech初创公司，其背后的资本主要来自保险公司
    - 比如，美国知名互联网保险公司Metromile的主营业务是车险、其五千万美金的投资其实来源于中国太平洋保险

保险科技对保险生态的影响

- 其次，保险中介机构受到了较大冲击
  - 大量的保险比价平台的出现
    - 使保险中介机构未来的生存空间越来越小
  - 投保人可不需要中介
    - 而仅靠人工智能来给他们提供保险服务

- 再者，众多的其他行业巨头被吸引进来
  - 百度、阿里巴巴、京东、腾讯都在布局保险行业，而沃尔沃等传统车企也开始提供保险服务

保险企业统计

细分领域	企业数量(家)	融资规模(亿美元)	平均融资规模(亿美元/家)
车险	114	65	0.570
企业险/商业险	92	8.4	0.091
健康/旅游保险	108	90	0.833
寿险/家财险	89	68	0.764
物品险	27	4.18	0.155
再保险	28	7.59	0.271
保险比价平台	280	11	0.039
员工福利平台	49	11	0.224
保险获客渠道	75	3.25	0.043
P2P保险	30	0.85	0.028
保险管理平台	57	3.7	0.065
保险数据/智能	100	26	0.260
保险基础设施/后端支持	222	10	0.045
保险教育/资源	31	0.53	0.017
新媒体	–	–	–
合计	1302	309.5	0.238

摘自《中国保险科技发展报告(2017)》
infzm.com

李伯根 | 制图

保险类型与模式创新

- 目前保险企业数
  - 1300多家
    - 初创公司
    - 或者保险科技企业
- 企业分类
  - 四大类型

产品创新

- 对车险定价和针对创新企业的商业险，譬如，在共享经济下，如何解决保险问题
  - 共享新能源汽车
  - 共享单车
  - 滴滴单车

- Airbnb和保险公司合作，向房主提供财产保险
  - 解决了房屋在预订住宿期间可能产生的损失问题

保险营销

- 保险比价

  - 传统保险属于非标准化设计，保单的价格差异体现了风险差异，但个人很难进行比价

  - 而通过大数据的应用，平台可以对一些原本难以量化的数据进行比较

企业内保险管理平台

- 目前在香港或海外的居民平均有5张以上的保单
  - 这些保单可能来自不同的保险公司
  - 客户很难通过单个保险平台综合管理所有保险产品
- 未来可能出现一款App能够实现个人保单统一管理
  - 把个人名下所有保险合同条款导入
  - 帮助客户管理各种保险风险，打破公司间的壁垒
- 多国社保账户
  - 美国与中国平台的某些功能互联互通

保险数据和智能平台

- 大数据公司想要进入保险业的切入口
  - 服务包括
    - 保险基础设施
    - 后端支持
    - 云计算
- 数据公司进入保险业最大的壁垒
  - 在于专业
  - 既要懂科技还要懂保险

信息互动　增强风控

- 信息互动
  - 资金池与服务系统的实时交互
  - 运营公司与服务系统的实时交互
  - 回报、权益、收益与监督交互
  - 与其它互金系统的交互

- 保险类型
  - 汽车保险
    - 驾驶状况、手机定位与租赁的实时互动
    - 新能源汽车的发展与演化
  - 航空险
    - 航空公司和系统服务的联通
    - 进机与进港的实时互动
  - 健康险
    - 健康状况的实时监护

平台演化 创新与发展

- 全球互联互通
  - 多国社保退休系统国际对接
- 一带一路互联互通
  - 新模式
  - 新产品
  - 新风险
- 其他新产品对接
  - 新风险分析模块

大数据与云系统之发明（1996-1998）

记忆存储一佰兆，

技术落后不能用，

万阳合伙造云台，

当今市值数万亿！

数据发展趋势

该图来源于书圈

期权与保险合约之博弈

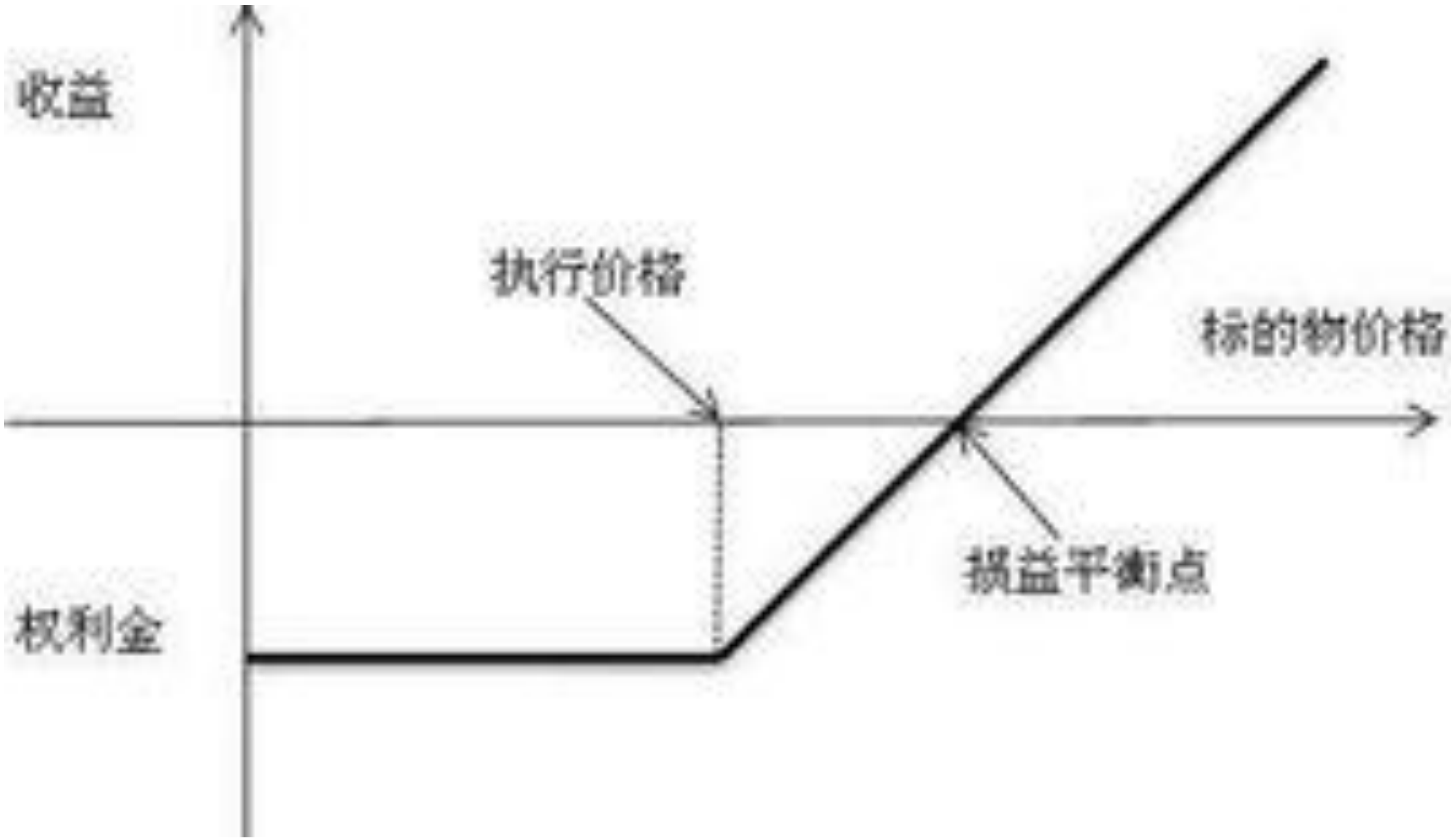
基于大数据、云计算与区块链的智能合约

期　　权

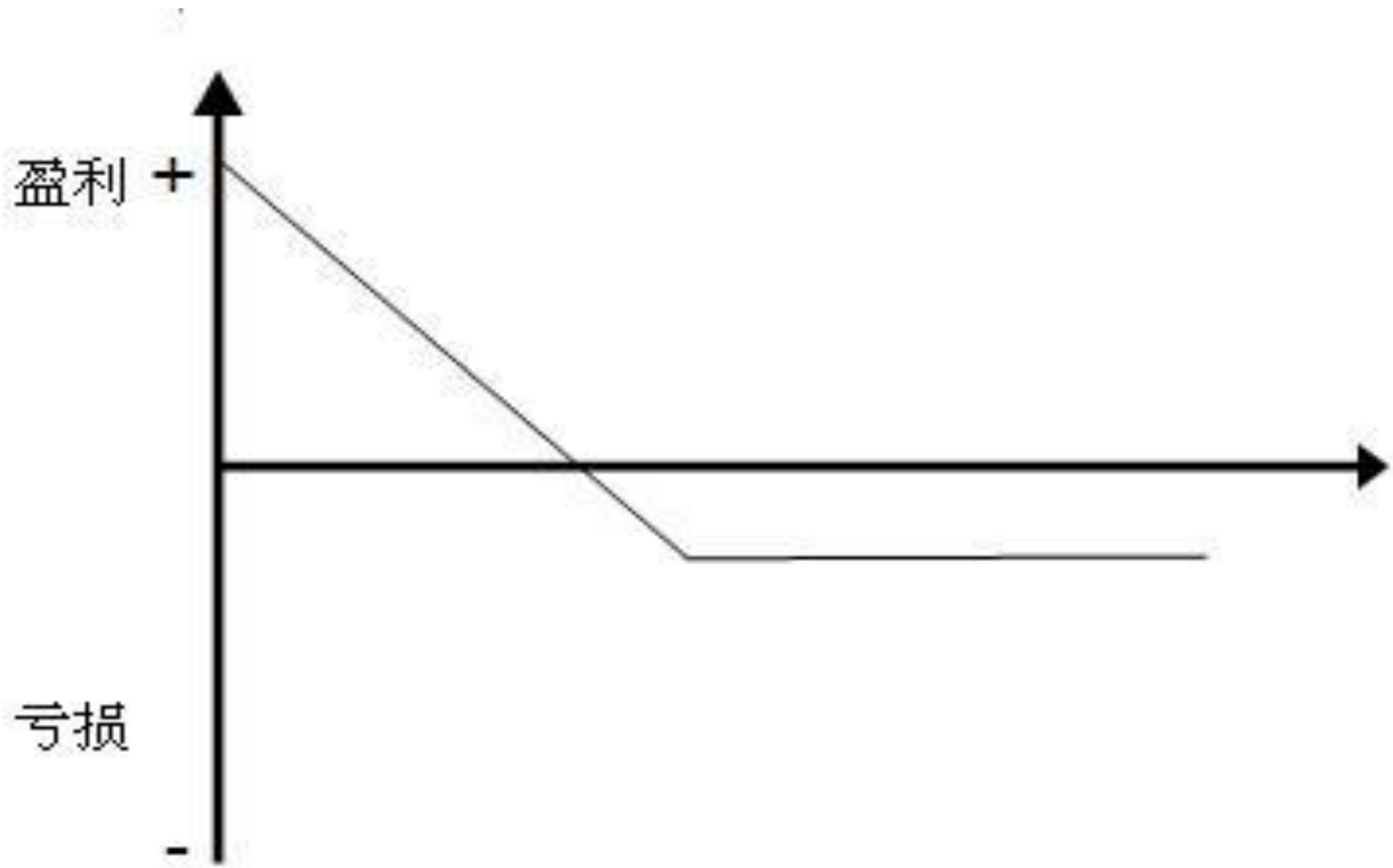
- 期权类型
  - 欧式与美式期权
    - 买方（Call，看涨期权）：(X-C)+
    - 卖方（Put，看跌期权）　：(C-X)+
  - 亚式与俄式期权

- 期权是零和赛局，买方与卖方权利与义务刚好相反
  - 对于买方
    - 似呼：“享有权利却不必负担义务，获利空间无限而风险有限”
    - 不过：　要付权利金（Premium），承担风险
  - 对于卖方
    - 似呼：“有义务而没有权利，风险无限却获利有限”
    - 不过：　可以合理进行保费定价，减少风险

买入期权



卖出期权



保险合约

- 卖权具备保险合约的特性
  - 当目标资产价值受损时
    - 卖权的买方（保户）会得到卖方（保险公司）的赔偿
    - 保费是卖方承担风险的报酬
- 具体一例
  - 下表列出股票卖权（Stock Put）与保险合约的相对关系。

卖权与合约之关系

表 1 股票卖权与保险合约的相对关系

保险合约	卖权
房屋价值	股价
保险合约期限	期权合约期限
到期前任何一天都适用	美式合约（到期前都可履约）
保险金额	执行价格
利率	利率、股利率
风险与发生事故机率	标的物的波动率
保费	权利金

BigData and Intelligent Cloud Platform vs. InsurTech and InsurBlockchain

Wanyang Dai

**Department of Mathematics and State Key Laboratory of Novel Software Technology
Nanjing University, China**

Jiangsu FinTech Research Center

**http://maths.nju.edu.cn/~wydai/
nan5lu8@nju.edu.cn**

**Invited Talk at Jiangsu Insurance Society
Nanjing, China**

June 12, 2017

Outline of Presentation

- InsurTech
  - Concept and status
- BigData (flows)
  - Popular definition and Real-world practice
  - Mathematical and statistical modeling
- Cloud-computing services
  - Concept, platform, and architecture
- Blockchain
  - Data structure, manager, and applications
- Artificial intelligence
  - Definition, status, and applications
- Resource allocation and insurance policy
  - Nash equilibrium, Fairness, and Pareto optimality
  - Dividend and portfolio
  - Game: Stock/real estate option and InsurContract

Models of Big Data Flow Dynamics

- **Models**
  - **Queueing networks and SDEs/SPDEs**
    - **with jumps and/or skew reflections**
    - **Forward and backward**
  - **Statistical view: functional (e.g., central) limits**
- **Machine learning system**
  - **High-dimensional time-space regression/series model**
  - **High-dimensional data analysis/analytics and processing**
  - **Discretized high-dimensional SDEs/SPDEs**
  - **with machine learning and intelligent control**

Time-Space BigData (I)

- **Consider a second-order B-SPDE**

$$V(t,x) = H(x) + \int_t^T b(s)\frac{\partial^2 V(s,x)}{\partial x^2}ds - \int_t^T \bar{V}(s,x)dW(s)$$

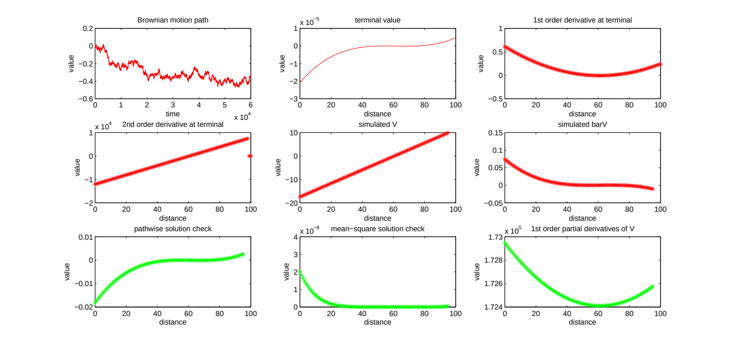
- **where, for given constants C and D**

$$H(x) = CW^2(T)\left(x - \frac{5.5D}{10}\right)^2\left(x - \frac{7.5D}{10}\right)$$

- **For each $s \in [0,T]$, $b(s)$ is either known or unknown parameter to be estimated**

Time-Space BigData (II)

- If $b(s)$ is known (e.g., $b(s) \equiv 1$), B-SPDE is solved



- If $b(s)$ is unknown, time-space regression models can be formulated

Popular Definition of BigData

- BigData is concerned with data sets
  - that are so large or complex that their sizes
  - beyond the ability of commonly used software tools
  - to capture, curate, manage, and process data
  - within a tolerable elapsed time
    - Snijders et al. (2012)
- The size of BigData is a constantly moving target
  - Dai et al. (100M, 1996-1998)
  - Economist (2011)
- Originality/development of BigData/cloud system

The Trend of Big Data

This graph is adapted from Shuquan

Latest 3-Dim Definition of BigData

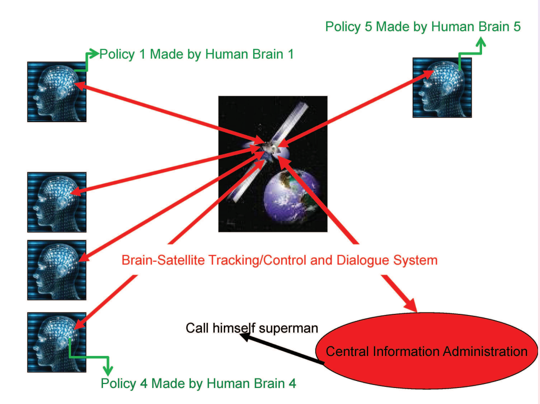
- 3-dim statistical characteristics with respect to processing capacity
  - High-volume amount of instantaneous real-time data
  - High-velocity of data in and out
  - High-variety range or dimension of data types and sources
    - De Mauro et al. (2016)
- New added feature
  - All history data
- How to quantify it statistically or mathematically?
  - Triply stochastic renewal reward processes
  - Lévy driven compound processes

BigData in InsurTech

- Batch arrivals of claims
  - Call intelligent decision engines
- Online data flows from Internet of Things
  - Car status monitoring
  - Health status monitoring
  - Fight traveling status monitoring
  - House status monitoring
  - Factory status monitoring
  - etc.
- Life growth credit data

Internet of Human Beings (I)

- **The system**



- **Dai (2009, IEEE Proceedings)**

Internet of Human Beings (II)

- It is a wireless communication network system
  - That directly links man's intelligent brain neuron nets (without computer added smart biochips or other added smart devices) to wireless Internet (typically a satellite communication system).
  - Through the system, the wireless Internet can read man's mind, track man's movement, control man's body functionality, etc.
  - Furthermore, the wireless Internet and man's mind can conduct dialogue each other directly through the system.
- Source link

http://maths.nju.edu.cn/~wydai/WanyangDaiHtmlC/pollutionC.html

Triply Stochastic Renewal Reward Process

- **Let τ_n be the jump times in terms of FS-CTMC $\alpha(\cdot)$**

$$\tau_0 \equiv 0,\ \tau_n \equiv \inf\{t > \tau_{n-1} : \alpha(t) \neq \alpha(t^-)\}$$

- **A process $A(\cdot)$ is called an TSRRP if, during each random time interval $[\tau_n, \tau_{n+1})$**
  - $A(\tau_n + \cdot)$ **is the counting process corresponding to a (conditional) delayed renewal reward process with**
  - **arrival rate $\lambda(\alpha(\tau_n))$ and mean reward $m(\alpha(\tau_n))$**
  - **squared coefficient of variation $\alpha^2(\alpha(\tau_n)) \in (0, \infty)$ and $\zeta^2(\alpha(\tau_n))$ (variance divided by the mean square)**
- **History data field: $\mathcal{F}_t = \sigma\{A(s), \alpha(s),\ s \leq t\}$**
- **Examples: MMPP and ON/OFF process**

FS-CTMC

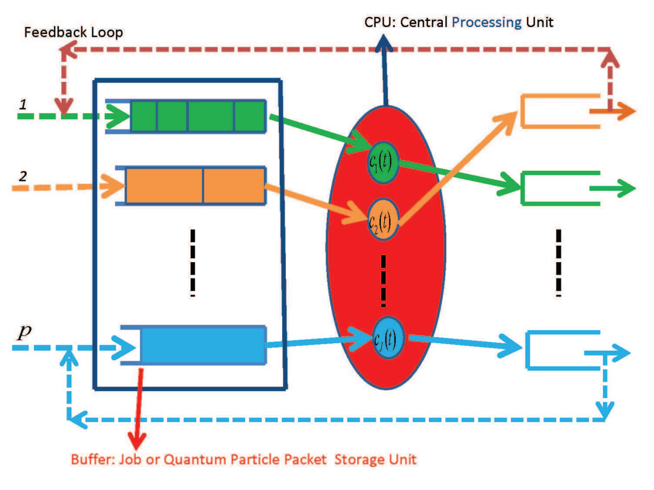
- **A stationary FS-CTMC** $\alpha = \{\alpha(t), t \in [0, \infty)\}$
  - **takes value in** $\mathcal{K} \equiv \{1, ..., K\}$
  - **generator matrix** $G = (g_{il})$ **(**$i, l \in \mathcal{K}$**)**

$$g_{il} = \begin{cases} -\gamma(i) & \textbf{if } \; i = l, \\ \gamma(i) q_{il} & \textbf{if } \; i \neq l \end{cases}$$

    - $\gamma(i)$ **is the holding rate for the chain in state** $i \in \{1, ..., K\}$
    - $Q = (q_{il})$ **is the transition matrix of its embedded discrete time Markov chain**

- **A special form of Lévy processes**

Single Pool Parallel-Server Center



Lévy Processes (I)

- $(\Omega, \mathcal{F}, P)$**: a probability space on which, define**
  - **A standard d-dim Brownian motion**
    - $W \equiv \{W(t), t \in [0, T]\}$
    - $W(t) = (W_1(t), ..., W_d(t))'$
  - **A h-dim Lévy process (or subordinator)**
    - $L \equiv \{L(t), t \in [0, T]\}$
    - $L(t) \equiv (L_1(t), ..., L_h(t))'$
- **Defining a filtration by $\{\mathcal{F}_t\}_{t \geq 0}$ with**

$$\mathcal{F}_t \equiv \sigma\{\mathcal{G}, W(s), L(\lambda s) : 0 \leq s \leq t\}$$

  - $\mathcal{G}$ **is σ-algebra independent of W and L**
  - W**,** L**, their components are independent each other**
  - $L(\lambda s) = (L_1(\lambda_1 s), ..., L_h(\lambda_h s))'$**,** $\lambda = (\lambda_1, ...\lambda_h)' > 0$

Lévy Processes (II)

- **Poisson random measure**

$$N_i((0,t] \times A) \equiv \sum_{0<s\leq t} I_A(L_i(s) - L_i(s^-))$$

  - $I_A(\cdot)$ **be the index function over the set** A
  - ν_i **for each** $i \in \{1, ..., h\}$ **be a Lévy measure**

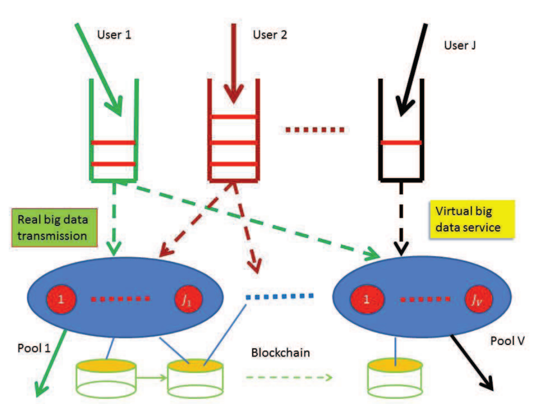
- **Centered process**

$$\tilde{N}(\lambda ds, dz) = (\tilde{N}_1(\lambda_1 ds, dz_1), ..., \tilde{N}_h(\lambda_h ds, dz_h))'$$

$$\tilde{N}_i(\lambda_i ds, dz_i) = N_i(\lambda_i ds, dz_i) - \lambda_i ds \nu_i(dz_i)$$

BigData-Blockchain and Cloud-Computing

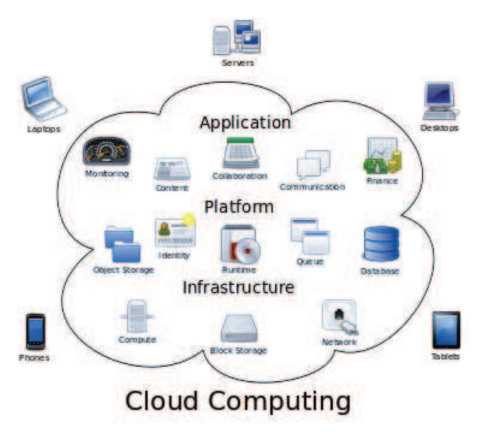
- **The multiple intelligent service pools**



- **The Fourth Industrial Revolution**

Supercomputer

Cloud-Computing



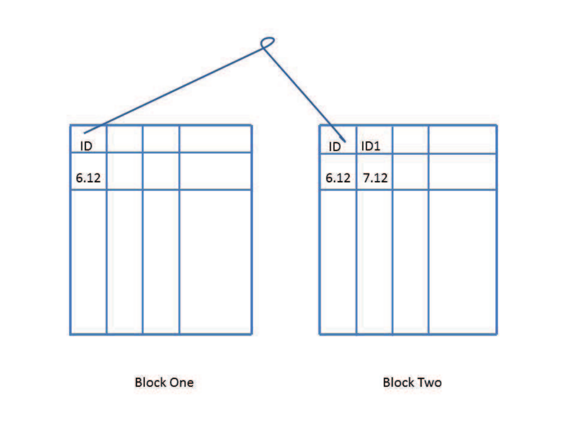
Adapted from Wiki

Cloud-Computing Based Services

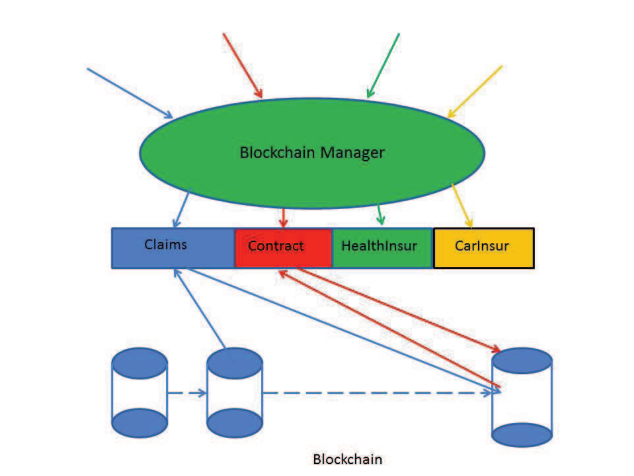
- Services in Information Era
  - DaaS (Data as a Service)
  - SaaS (Software as a Service)
  - PaaS (Platform as a Service)
  - IaaS (Infrastructure as a Service)
- Blockchains based services in Era of Internet+
  - CommTech
  - FinTech
  - InsurTech
  - TaxTech
  - HealthTech
  - EnergyTech
  - PowerTech
  - AgricultureTech

Block Tables in Blockchain

- **Correlated database block tables via a link**



Blockchain Manager by Intelligent Engines



Definition of Blockchain

- A distributed database that is used to maintain
  - a continuously growing list of records, called blocks
    - Each block contains a timestamp
    - and a link to a previous block
- Typically managed by a peer-to-peer network
  - collectively adhering to a protocol
  - for validating new blocks
- By design, blockchains are inherently resistant to modification of the data
  - Once recorded, the data in any given block cannot be altered retroactively without the alteration of all subsequent blocks and the collusion of the network

Functionality of Blockchain

- **A blockchain can serve as**
  - **an open and distributed ledger**
  - **that can record transactions**
    - **between two parties efficiently**
    - **in a verifiable and permanent way**
- **The ledger itself can also be programmed**
  - **to trigger transactions automatically**

Advantages of Blockchain

- Blockchains are secure by design
  - An example of a distributed computing system
    - with high Byzantine fault tolerance
  - Decentralized consensus can be achieved with a blockchain
- This makes blockchains potentially suitable for
  - the recording of events
  - medical records
  - records management activities
  - identity management
  - transaction processing
  - documenting provenance

History and Applications of Blockchain

- The first blockchain was conceptualised in 2008 by Nakamoto and implemented in 2009
  - as a core component of the digital currency bitcoin
  - where it serves as the public ledger for all transactions
- The invention of the blockchain for bitcoin made it the first digital currency
  - to solve the double spending problem
  - without the use of a trusted authority or central server
- The bitcoin design has been the inspiration for other applications
  - e.g., in more insurance problems

Real BigData Services

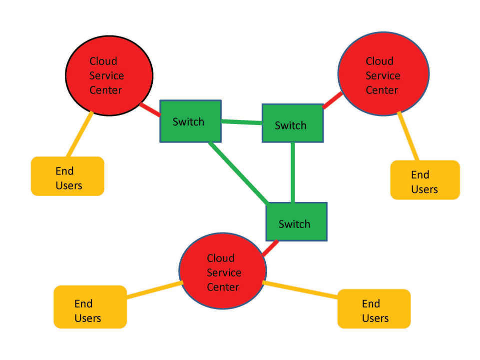
- **Smart particle storages for queued jobs**
  - **Huge number of real data packets (or batches of packets) to be transmitted over wireless channels or wireline links**
  - **Data storage to BlockChain or distributed database**
    - **Any data cluster is divided into batches of packets pre-assigned or optimally being assigned to different sub-databases**
- **Applications in**
  - **Communication systems**
  - **Cloud-computing based video-on-demand services**
  - **Internet of Things**
  - **Internet of Human Beings**

Virtue BigData Services

- Smart particle storages for queued jobs
  - Short data messages to indicate cloud-computing based huge amount of data service requirements
  - Resources-consuming, expensive costs, high complexities
- Querying Blockchain, running intelligent engine
  - InsurTech & FinTech (e.g., credit evaluation for risk prediction and portfolio hedging), network control
    - Dai (2011, 2015, 2013, Operations Research, etc.)
- J.P. Morgan of time priority: ps vs. 360m traders
  - Central Bank of China: digital finance
  - China Merchants Bank, Agriculture Bank with Ali Cloud
- Handing health-care processes
  - e.g., gene sequence comparison and diagnosis by mutual information techniques in Goldsmith et al. (2003)

Future Network Architecture

- **The Architecture (Dai (2006, 2014))**



- **The invention of cloud system**
  - **Supercomputer, cloud computing and platform**

The Aims of Studying CC (I)

- **To meet the requirements for**
  - **large-scale data storage, processing, mining, and services**
- **How effectively use history data in BlockChain?**
  - **Security (adm) checking with limited data requirement**
  - **Decision-making with BigData support**
    - **Kalman or Wiener Filter? Machine-learning?**
- **Optimal or reasonable (delay, loss) performance**
  - **Product-form solutions for integrated services packet networks and cloud computing systems**
    - **Proceedings of MICAI, IEEE CS Press (Dai, 2006)**
    - **Mathematical Problems in Engineering (Dai, 2014)**
  - **Diffusion approximations for multiclass queueing networks under preemptive priority service discipline**
    - **Applied Mathematics and Mechanics (Dai, 2007)**

The Aims of Studying CC (II)

- Maximizing profit
  - Optimal control with monotonicity constraints for a parallel-server loss channel serving multi-class jobs
    - Mathematical and Computer Modeling of Dynamical Systems (Dai, 2014)
- Maximizing the utility of resource allocation while minimizing the operational cost of the system simultaneously
  - Optimal Rate Scheduling via Utility-Maximization for J-User MIMO Markov Fading Wireless Channels with Cooperation
    - Operations Research (Dai, 2013)

The History of CC

- First product using platform, architecture, network, software as a service in a Plug-in and Play way
  - SimNet (1996.7-1998, AT&T (now Nokia) Bell Labs)
    - for telecommunication network design, performance evaluation, pricing, tariff, and strategy planning
    - Dai (principal investigator and developer), etc.
    - AT&T (now Nokia) Bell Labs multi-million US dollars and multi-organizations "lightning" R&D project
    - won "Technology Transfer"
  - Listening Post (1997-1998, Bell Labs)
    - Dai (independent investigator and developer)
- Earliest studies of cloud-computing systems
  - via stochastic processing networks, reflecting diffusions, and queueing systems
  - Whitt (1970's, Bell Labs), Harrison (1970's, Stanford), Dai (1992-96, Georgia Tech), etc.

Definition of Artificial Intelligence

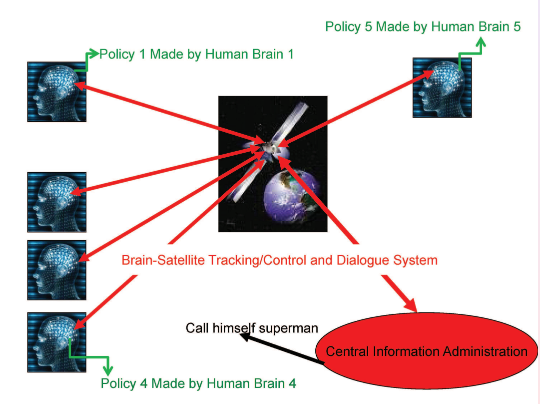
- **AI is intelligence exhibited by machines**
- **In computer science, the field of AI research defines itself as the study of "intelligent agents": any device that**
  - **perceives its environment and takes actions that**
  - **maximize its chance of success at some goal**
- **The term "artificial intelligence" is applied**
  - **when a machine mimics "cognitive" functions**
  - **that humans associate with other human minds**
    - **such as "learning" and "problem solving"**

Goals and Approaches of AI

- The central problems (or goals) of AI include
  - reasoning, knowledge, planning, learning
  - natural language processing (communication)
  - perception/the ability to move and manipulate objects
- Approaches include
  - statistical methods
  - computational intelligence
  - traditional symbolic AI
- Tools in AI include
  - versions of search and mathematical optimization
  - logic, methods based on probability and economics
- The AI field draws upon
  - computer science, mathematics, psychology, linguistics,
  - philosophy, neuroscience, artificial psychology, etc.

Status and Applications of AI (I)

- **Internet of Human Beings**



- **Dai (2009, IEEE Proceedings)**

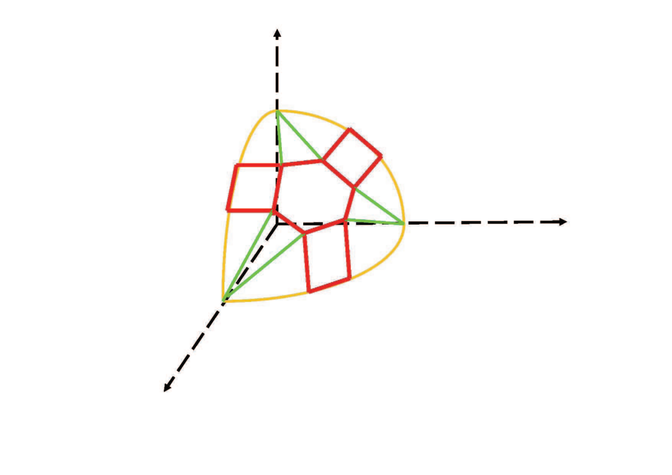
Status and Applications of AI (II)

- Telecommunication networks
  - Design, performance evaluation, pricing, and tariff
  - The invention of the first product of cloud-computing platform and system (1996-1999)
- Smart grid
  - International Conference of Energy and Power, Toronto
- Competing at a high level in strategic game systems, e.g., Chess and Go (AlphaGo)
- Self-driving cars
- Intelligent routing in content delivery networks
- Military simulations
- Interpreting complex data

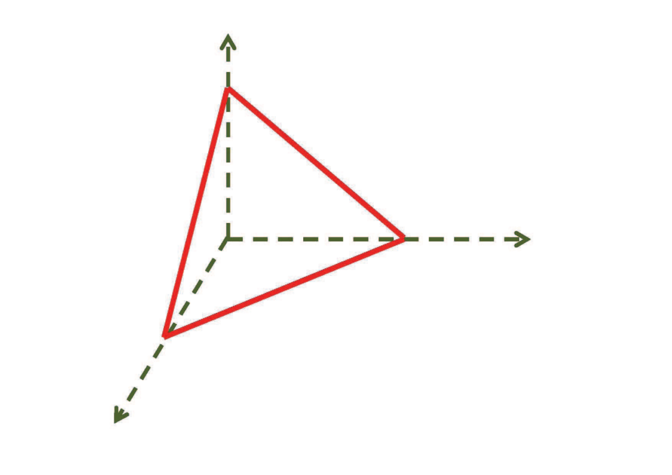
Resource Allocation and Insurance Decision

- **Resource allocation**
  - Rate and power scheduling
    - Operations Research (Dai, 2013)
  - ATO supply chain scheduling
    - Dai and Jiang (2007)
    - Applications in P2P Net Loan
  - Mean-variance Portfolio
    - Dai (2011, 2015)
- **Insurance decision**
  - Dividend and inject capital policy of beneficiaries
    - Harrison and Taylor (1978), Avram et al. (2007)
  - Who are the beneficiaries
    - P2P Net Loan and Others?
  - Game: Stock/real estate option and InsurContract
    - Dai (2011,2015), Liu and Dai (2010)

Service Capacity Region (I)



Service Capacity Region (II)



Queueing Signal and Particle Storage (I)

- **Signal or particle storage process S to be processed**
  - **can be considered as a queueing process**
    - **e.g., in a real communication network system**
  - **denoted by $S(\cdot) = (S_1(\cdot), ..., S_p(\cdot))'$**
    - $S_i(t)$ **is the number of ith class job packets**
    - **stored in the ith buffer for $i \in \{1, ..., p\}$ at time t**
- **Queueing dynamics of the network**

$$S(t) = S(0) + A(t) - D(t)$$

  - **The ith component $A_i(t)$ of $A(t)$**
    - **the total number of jobs arrived to buffer i by time t**
  - **The ith component $D_i(t)$ of $D(t)$**
    - **the total number of jobs departed from buffer i by time t**

Queueing Signal and Particle Storage (II)

- Statistical characterizations of queueing signals
  - Lévy driven processes
    - as solutions to SDEs with Lévy jumps
  - Reflecting Brownian motions (RBMs)
    - Approximating models under certain conditions
    - e.g., for general renewal or renewal reward input signals
- How to handle the gaps between
  - The achievable maximal capacity
    - corresponding to the assumed distributions of transmitted signals under certain coding techniques
    - e.g., BMs or Gaussian signals
  - The required one
    - corresponding to real queueing signals
    - e.g., RBMs or Lévy driven non-Gaussian signals

Insurance Surplus Model

- **Cramér-Lundberg model**

$$S(t) = S(0) + dt - \sum_{k=1}^{N(t)} C_k$$

- **Extended beneficial model**

$$U(t) = S(t) - \sum_{j=1}^{J} L_j(t)$$

- **Beneficial model with inject capital**

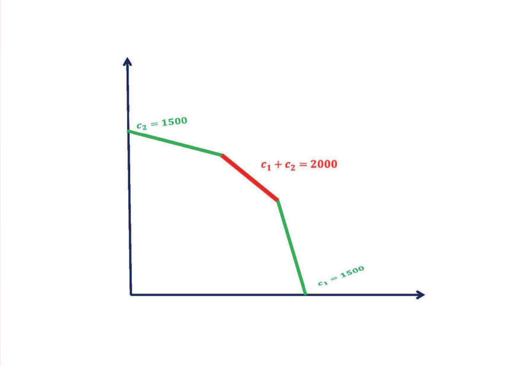
$$V(t) = U(t) + \sum_{j=1}^{J} R_j(t)$$

Example

- **To conduct performance and numerical comparisons between our designed game and linear-utility based scheduling policies**
    - **by simulating their corresponding RDRSs and solving the related game problems**
- **we take $V = 1$, $J = 2$, and $\alpha(t) \equiv 1$ for all $t \in [0, \infty)$**

Capacity Region for the Example

- **Capacity region is assumed to be a non-degenerate with $\rho_1^2 = \rho_2^1 = 1500$ and upper bound satisfying $c_1 + c_2 = 2000$, i.e.,**



Utility Functions

- **Utility functions for user 1 and user 2**

$$U_1(q_1, c_1) = q_1 \ln(c_1), \quad U_2(q_2, c_2) = -\frac{q_2^2}{c_2^2}$$

  - **which correspond to the proportionally fair and minimal potential delay allocations respectively**

- **Based on these utility functions, we can design our rate-scheduling policy at each time point $t \in [0, \infty)$ by a Pareto maximal-utility Nash equilibrium point to the non-zero-sum game problem (Dai (2013))**

Utility-Maximization Game-Scheduling Policy

- **The policy (Operations Research, Dai (2013))**

$$\max_{c\in\mathcal{R}} U_j(q,c), \quad j\in\{0,1,2\}, \quad q\in R_+^2$$

where, $U_0(q,c) = U_1(q,c) + U_2(q,c)$

- **If** $c^* = (c_1^*, c_2^*)$ **is a solution to the game problem**

$$\begin{aligned} &U_0(q,c^*) \geq U_0(q,c), \\ &U_1(q,c^*) \geq U_1(q,c_{-1}^*) \quad \textbf{with} \quad c_{\cdot-1}^* = (c_1, c_2^*) \\ &U_2(q,c^*) \geq U_2(q,c_{-2}^*) \quad \textbf{with} \quad c_{\cdot-2}^* = (c_1^*, c_2) \end{aligned}$$

- **i.e., if a game player's rate service policy is unilaterally changed, his utility can not be improved**

Dual-Cost Functions

- **In a real-world system, the parameter vector q is the randomly evolving queue length process $Q(t)$**
- **How to evaluate the effectiveness of the myopic service policy designed through the game problem globally over the whole time horizon $[0, \infty)$ is our major concern**
- **To reach this goal, we first identify the associated dual-cost functions for $U_j(q, c)$ with $j \in \{1, 2\}$ as**

$$C_1(q_1, c_1) = \frac{q_1^2}{2\mu_1 c_1}, \quad C_2(q_2, c_2) = \frac{2q_2^3}{3\mu_2 c_2^3}$$

Dual-Cost Non-Zero-Sum Game Problem

$$\min_{q \in R_+^2} C_j(q,c) \text{ subject to } \frac{q_1}{\mu_1} + \frac{q_2}{\mu_2} \geq w$$

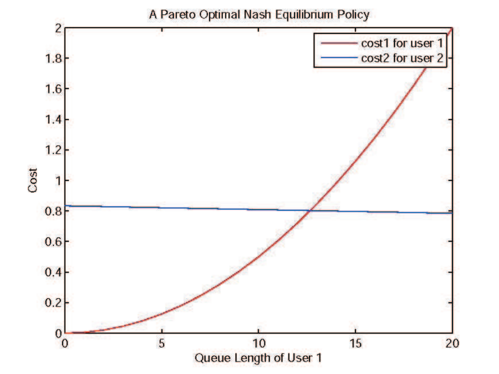
- **for a fixed constant $w > 0$, a fixed $c \in \mathcal{R}$, and all $j \in \{0,1,2\}$ with $C_0(q,c) = C_1(q,c) + C_2(q,c)$**
- **If $q^* = (q_1^*, q_2^*)$ is a solution to the game problem**

$$\begin{aligned} C_0(q^*,c) &\leq C_0(q,c), \\ C_1(q^*,c) &\leq C_1(q_{-1}^*,c) \quad \text{with} \quad q_{-1}^* = (q_1, q_2^*), \\ C_2(q^*,c) &\leq C_2(q_{-2}^*,c) \quad \text{with} \quad q_{-2}^* = (q_1^*, q_2). \end{aligned}$$

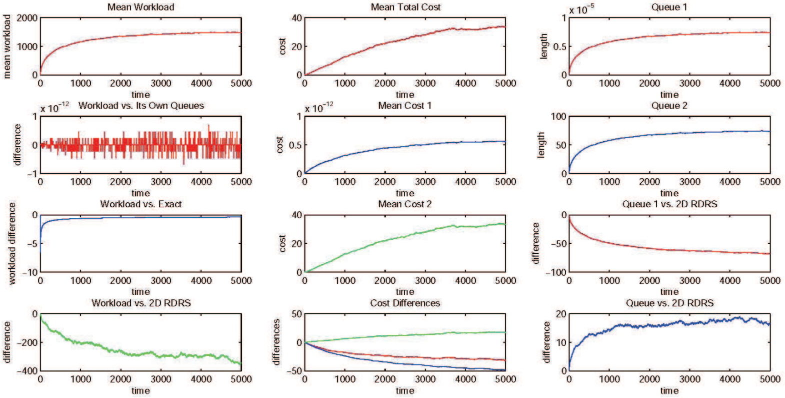
- **i.e., if a player j with $j \in \{1,2\}$ unilaterally changes his bid's policy q_j, his cost can not be reduced**

Pareto Dual-Cost Minimal Nash Equilibrium

- **The unique Pareto minimal Nash equilibrium policy $q^* = (q_1^*, q_2^*)$ q^* is the intersection point of the red and blue curves**



Numerical Simulation Comparisons



Game between Option and InsurContract

- Game: Stock/real estate option and InsurContract
  - Dai (2011,2015), Liu and Dai (2010)