On the traveling neuron nets (human brains) controlled by a satellite communication system

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***Abstract*—We deal with the stochastic modeling issue for a real existing Man-Machine (Brain-Satellite) communication system, which consists of traveling neuron nets (e.g., human brains) tracked and controlled by a satellite communication system. The neuron nets have the ability to pass their controlled functionality to other neuron nets via the satellite communication system. Basing on an established model, we derive the probability distribution for certain hitting time that represents the smallest time at which all of the neuron nets belonging to some particular biological group essentially lose their battle or competition capability.**

I. Introduction

With the release of recent achievements about how to identify natural images from human brain activity (see, e.g., Kay *et al* [7], Miyawaki *et al* [8], Taylor and Worseley [9]), the partial functionality of an existing Brain-Satellite dialogue communication system is disclosed. This system is a Man-Machine interactive and tracking system (see, e.g., Figure 1), which is known as *dream-taking* system by many people in

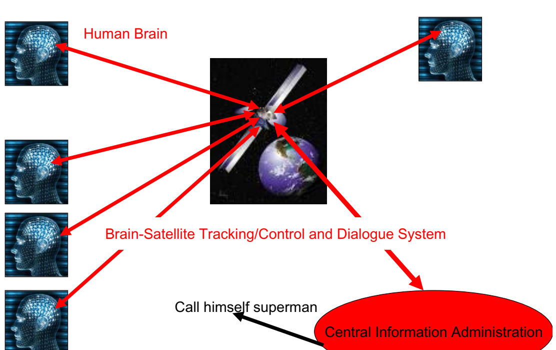

Fig. 1. A Brain-Satellite tracking/control and dialogue system

the public since it has been widely used for the interests of the nation who owns the system. Although there are some important issues needed to be settled urgently about how to use the system morally and legally in order not to violate intellectual property laws and not to improperly torture people, etc. via the system, we will only address the further functionality of the system technically in this paper. Interested readers are also directed to Ananasso [1], Johnannsen [6] and etc. for more details about satellite communication systems, Goodwin and Sin [5], Dai [3] and etc. for more details about wireless tracking systems, Andrew *et al* [2], Dai and Wang [4] for more details about wireless multiple access systems.

Concretely, we will study the modeling issue for a real existing system which comprises of neuron nets (e.g., human brains) whose functionality is controlled by a satellite communication system and who travel around certain region. For easy reference, we refer them as *controlled traveling neuron nets*. In the region, there is also other type of biological neuron nets who are refereed as *visitors* from some specific place (base station). Once a controlled traveling neuron net bumps into a visitor in the region, the visitor is infected to become a controlled traveling neuron net via the satellite communication system. After the infected visitor goes back to his home base station, all of the neuron nets in the station become controlled neuron nets, which represents that all of these neuron nets in the base station essentially lose their battle or competition capability. Hence, in this paper, we iteratively formulate the dynamical evolution of the system for both simulation and analytical resolution studies, and moreover, the formulated model is employed to derive the probability distribution for certain hitting time that represents the smallest time at which there is a controlled traveling neuron net bumping into a visitor in the region. Generalization of the above system to network environment with multiple regions and multiple base stations will also be addressed.

The rest of the paper is organized as follows. In Section II, we present our system model, and conduct simulation case studies. In Section III, we handle analytical resolution issue and provide criteria for simulation code verification. Conclusion of this paper is given in IV.

II. Modeling the System

The system under consideration consists of a region and a base station (see Figure 2 for such an example). For the region, there is an external arrival stream of controlled traveling neuron nets, which associates with a sequence of arrival batches with deterministic interarrival times and random sizes $\{E(i), i = 0, 1, 2, ...,\}$ with mean value b, where i is the index of the ith arrival batch. Each arrival traveling neuron net denoted by $j \in \{0, 1, 2, ...,\}$ to the region will stay there

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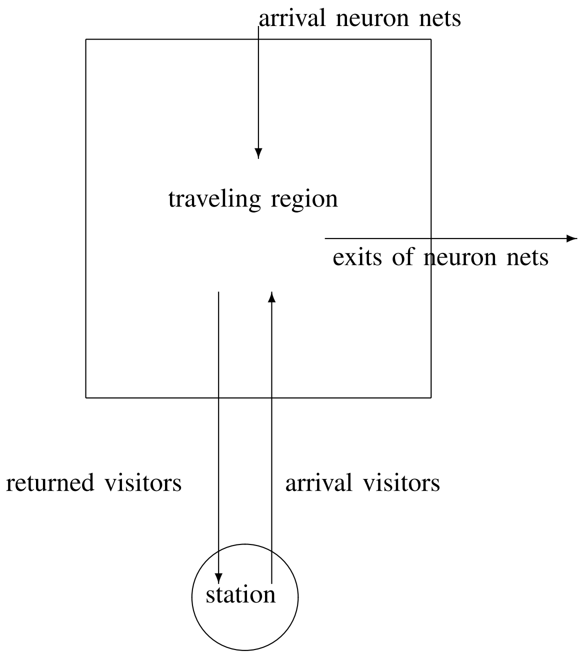

Fig. 2. A system with single region and single base station

for a random amount of time $v(j)$ with mean value $1/\beta$. For the region, there is also a sequence of visitor batches from the base station with deterministic interarrival times and random sizes $\{\bar{E}(i)\}$ with mean $\bar{b}$. Each arrival visitor $j \in \{0, 1, 2, ...\}$ will stay in the region for a random amount of time $\bar{v}(j)$ with mean value $1/\bar{\beta}$. Here, we assume that visitors' arrivals and departures are independent of traveling neuron nets' activities. Once a traveling neuron net in the region bumps into a visitor at some point of the region with certain (bumping) probability during their stay in the region, the visitor will become a controlled traveling neuron net, where we will employ the assumption that the unit bumping probability h is given, which represents the average meeting possibility of a traveling neuron net and a visitor who visit the region at the same time unit. Here we remark that the bumping probability can be one, for example, if the base station is within the region, and in the meanwhile, if traveling neuron nets visits the base station as scheduled.

One of the most interesting problems related to the system is how to derive the probability distribution of the hitting time T which is the smallest time that there is a visitor from the base station who becomes a controlled traveling neuron net, that is,

$$T \equiv \inf\{t: \text{ there is a traveling neuron net who meets a visitor from the base station at some point of the region at time } t\}. \tag{2.1}$$

In order to get the distribution, one needs to model the dynamical evolution of the system properly for both simulation and analytical resolution studies. Thus, let $Q(t)$ be the number of traveling neuron nets and $\bar{Q}(t)$ be the number of visitors in the region at time t, then we have the following iterative formula, for $t \in [0, 1)$,

$$Q(t) = E(0), \quad \bar{Q}(t) = \bar{E}(0), \tag{2.2}$$

and for $t \in [k, k+1)$ with $k \in \{1, 2, ...\}$,

$$Q(t) = Q(k-1) + E(k) - D(k), \tag{2.3}$$

$$\bar{Q}(t) = \bar{Q}(k-1) + \bar{E}(k) - \bar{D}(k) \tag{2.4}$$

where $D(k)$ is the total number of departed traveling neuron nets after finishing their stay in the region or dropping by the region during time interval $(k-1, k]$ and it is counted at the end of the unit time interval; $\bar{D}(k)$ is the corresponding total number of departed visitors during $(k-1, k]$ and it is also counted at the end of the time interval.

From the iterative formula (2.2) and (2.4), one can conduct simulation study and get the simulated probability distribution for T in (2.1). To do so, we use the smallest nonnegative integers satisfying (2.1) to approximate T and get related distribution. In the below examples, we derive the distributions with different bumping probabilities, $1/40$ (the red one) and $1/100$ (the blue one), respectively. Other parameters are taken as follows: $b = 5$, $\beta = 2$, $\bar{\lambda} = 5$ and $\bar{\beta} = 2$. In the simulation, we repeatedly run our program 420 times to get the reasonable accuracy. During the simulation, we used the below formula which represents the conditional probability that there are i departures at time $k+1$ if $Q(k) = j$ and the staying time for each arrival neuron net is exponentially distributed

$$C_j^i (1 - e^{-\beta})^{j-i} e^{-i\beta}.$$

Similar explanation applies to the process $\bar{Q}(\cdot)$ and the simulation results are displayed in Figure 3.

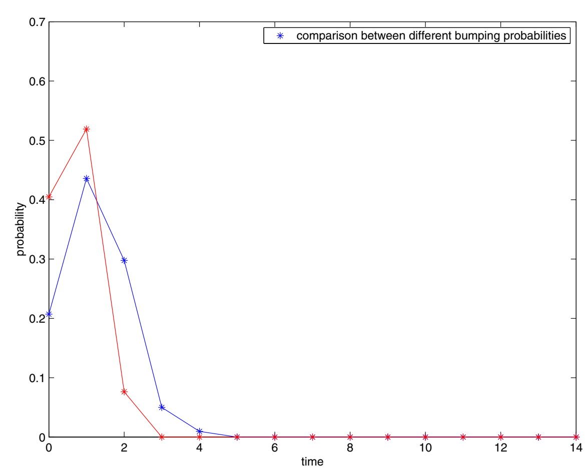

Fig. 3. Probability distributions of the approximated hitting times

III. Analytical Resolution

For the purposes of more accurate study and simulation code verification, we derive the analytical solution of the hitting probabilities for T. Let $A_{[t_1,t_2)}$ denote the below event for any two nonnegative constants t_1 and t_2,

$$A_{[t_1,t_2)} \equiv \{\text{There is a traveling neuron net in the region who meets a visitor from the base station during the time interval } [t_1,t_2)\}, \tag{3.5}$$

and $\bar{A}_{[t_1,t_2)}$ be the corresponding complementary event. By assumptions, we know that $Q(t)$ and $\bar{Q}(t)$ have the Markovian and independent increment properties, moreover, the event $A_{[k,k+1)}$ for $k \in \{0,1,...\}$ depends only on $Q(k)$ and $\bar{Q}(k)$. Thus, for the given $a=0$, we have,

$$P\{T=0\} = \sum_{i_0=1}^{\infty}\sum_{j_0=1}^{\infty}\left(1-(1-h)^{i_0 j_0}\right) P\{E(0)=i_0\}P\{\bar{E}(0)=j_0\}, \tag{3.6}$$

and for any given nonnegative integer $a \geq 1$, it follows from (2.2) and the total probability formula that

$$\begin{aligned}
& P\{T=a\} \\
= \; & P\{\bar{A}_{[0,1)}, \bar{A}_{[1,2)}, ..., \bar{A}_{[a-1,a)}, A_{[a,a+1)}\} \\
= \; & (1-\sum_{i_0=1}^{\infty}\sum_{j_0=1}^{\infty}(1-(1-h)^{i_0 j_0}) \\
& \quad P\{E(0)=i_0\}P\{\bar{E}(0)=j_0\}) \\
& (1-\sum_{i_0=0}^{\infty}\sum_{j_0=0}^{\infty}\sum_{k_1=0}^{i_0}\sum_{l_1=0}^{j_0}\sum_{i_1=0}^{\infty}\sum_{j_1=0}^{\infty} \\
& P\{h,i_0,j_0,i_1,j_1,k_1,l_1\} \\
& I\{i_0-k_1+i_1>0\}I\{j_0-l_1+j_1>0\} \\
& P\{E(1)=i_1\}P\{D(1)=k_1/Q(0)=i_0\} \\
& P\{Q(0)=i_0\} \\
& P\{\bar{E}(1)=j_1\})P\{\bar{D}(1)=l_1/\bar{Q}(0)=j_0\} \\
& P\{\bar{Q}(0)=j_0\}) \\
& ... \; ... \; ... \\
& ... \; ... \; ... \\
& ... \; ... \; ... \\
& (1-\sum_{i_{a-2}=0}^{\infty}\sum_{j_{a-2}=0}^{\infty}\sum_{k_{a-1}=0}^{i_{a-2}}\sum_{l_{a-1}=0}^{j_{a-2}} \\
& \quad \sum_{i_{a-1}=0}^{\infty}\sum_{j_{a-1}=0}^{\infty} \\
& P\{h,i_{a-2},j_{a-2},i_{a-1},j_{a-1},k_{a-1},l_{a-1}\} \\
& I\{i_{a-2}-k_{a-1}+i_{a-1}>0\} \\
& I\{j_{a-2}-l_{a-1}+j_{a-1}>0\} \\
& P\{E(a-1)=i_{a-1}\} \\
& P\{D(a-1)=k_{a-1}/Q(a-2)=i_{a-2}\} \\
& P\{Q(a-2)=i_{a-2}\} \\
& P\{\bar{E}(a-1)=j_{a-1}\} \\
& P\{\bar{D}(a-1)=l_{a-1}/\bar{Q}(a-2)=j_{a-2}\} \\
& P\{\bar{Q}(a-2)=j_{a-2}\}) \\
& (\sum_{i_{a-1}=0}^{\infty}\sum_{j_{a-1}=0}^{\infty}\sum_{k_a=0}^{i_{a-1}}\sum_{l_a=0}^{j_{a-1}}\sum_{i_a=0}^{\infty}\sum_{j_a=0}^{\infty} \\
& P\{h,i_{a-1},j_{a-1},i_a,j_a,k_a,l_a\} \\
& I\{i_{a-1}-k_a+i_a>0\} \\
& I\{j_{a-1}-l_a+j_a>0\} \\
& P\{E(a)=i_a\} \\
& P\{D(a)=k_a/Q(a-1)=i_{a-1}\} \\
& P\{Q(a-1)=i_{a-1}\} \\
& P\{\bar{E}(a)=j_a\} \\
& P\{\bar{D}(a)=l_a/\bar{Q}(a-1)=j_{a-1}\} \\
& P\{\bar{Q}(a-1)=j_{a-1}\})
\end{aligned} \tag{3.7}$$

where $I\{\cdot\}$ is the indicator function over the set $\{\cdot\}$ and for $w \in \{1,...,a\}$,

$$\begin{aligned}
& P\{h,i_{w-1},j_{w-1},i_w,j_w,k_w,l_w\} \\
= \; & \sum_{u=0}^{(i_{w-1}-k_w+i_w)*(j_{w-1}-l_w+j_w)-1} \\
& C^1_{(i_{w-1}-k_w+i_w)*(j_{w-1}-l_w+j_w)-u}(1-h)^u h \\
& P_{(i_{w-1}-k_w+i_w)*(j_{w-1}-l_w+j_w)-u} \\
= \; & \sum_{u=0}^{(i_{w-1}-k_w+i_w)*(j_{w-1}-l_w+j_w)-1} (1-h)^u h \\
= \; & 1-(1-h)^{(i_{w-1}-k_w+i_w)*(j_{w-1}-l_w+j_w)}.
\end{aligned} \tag{3.8}$$

In (3.8), we used the assumption that all of the paired traveling neuron net and visitor have the equal probability to be the first pair who bump into together. In order to explicitly calculate the distribution given in (3.7), we need certain assumption on the distribution of random batch sizes for arrival controlled traveling neuron nets. Here we mainly discuss the case that: all of the batches have the same constant size b_m. Then we have that, for all $w \in \{0,1,...,\}$,

$$P\{E(w)=b\} = 1, \tag{3.9}$$

and for all $w \in \{0,1,...\}$, $j \in \{0,1,...\}$

$$P\{\bar{E}(w)=j\} = \bar{\lambda}^j e^{-\bar{\lambda}}/j! \tag{3.10}$$

The conditional departure distributions for traveling neuron nets and visitors at time $w \in \{1,2,...,a\}$ can be calculated below,

$$\begin{aligned}
& P\{D(w)=k_w/Q(w-1)=i_{w-1}\} \\
& = C^{k_w}_{i_{w-1}}(1-e^{-\beta})^{i_{w-1}-k_w}e^{-k_w\beta},
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
& P\{\bar{D}(w)=l_w/\bar{Q}(w-1)=j_{w-1}\} \\
& = C^{l_w}_{j_{w-1}}(1-e^{-\bar{\beta}})^{j_{w-1}-l_w}e^{-l_w\bar{\beta}}.
\end{aligned} \tag{3.12}$$

Concerning the probability distributions of $Q(w)$ and $\bar{Q}(w)$ for $w=0$, we have

$$P\{Q(0)=b\} = P\{E(0)=b\} \tag{3.13}$$

$$P\{\bar{Q}(0)=j\} = P\{\bar{E}(0)=j\}. \tag{3.14}$$

For $w = 1$, we need the assumption that arrivals at time 1 can not immediately leave the region at the time point, then it follows from (2.2)-(2.4) that, for $i_1 \in \{b, ..., 2b\}$,

$$\begin{aligned} & P\{Q(1) = i_1\} \\ & = P\{D(1) = 2b - i_1 / Q(0) = b\}, \end{aligned} \tag{3.15}$$

and moreover, we have

$$\begin{aligned} & P\{\bar{Q}(1) = j_1\} \\ & = \sum_{j_0=0}^{\infty} P\{\bar{Q}(0) = j_0\} \\ & \quad (\sum_{k_1=0}^{j_0} P\{\bar{D}(1) = k_1 / \bar{Q}(0) = j_0\} \\ & \quad P\{\bar{E}(1) = j_1 - j_0 + k_1\}). \end{aligned} \tag{3.16}$$

Similarly, for $w \in \{2, ..., a-1\}$, it follows from (2.2)-(2.4) that, for $i_w \geq b$,

$$\begin{aligned} & P\{Q(w) = i_w\} \\ & = \sum_{i_{w-1}=b}^{\infty} P\{Q(w-1) = i_{w-1}\} \\ & \quad P\{D(w) = i_{w-1} - i_w + b / \\ & \quad\quad Q(w-1) = i_{w-1}\} \end{aligned} \tag{3.17}$$

$$\begin{aligned} & P\{\bar{Q}(w) = j_w\} \\ & = \sum_{j_{w-1}=0}^{\infty} P\{\bar{Q}(w-1) = j_{w-1}\} (\sum_{k_w=0}^{j_{w-1}} \\ & \quad P\{\bar{D}(w) = k_w / \bar{Q}(w-1) = j_{w-1}\} \\ & \quad P\{\bar{E}(w) = j_w - j_{w-1} + k_w\}) \end{aligned} \tag{3.18}$$

Example 3.1: In the below numerical example presented in Table I, one can see that the simulation errors obey the general simulation rule, that is, in the order of $O(1/\sqrt{420})$, where we compare the simulation results obtained previously and the analytical solution derived in (3.6) and (3.7).

$b = 5$, $\beta = 2$, $\lambda = 5$, $\beta = 2$			
h	a	Simulated	Analytical
1/40	0	0.4048	0.4482
1/40	1	0.5190	0.4732
1/100	0	0.2071	0.2173
1/100	1	0.4357	0.4419

TABLE I
COMPARISONS OF THE HITTING TIME PROBABILITIES FOR T

In the end, we comment that the previous discussion can be readily extended to other distribution cases, for example, random batch sizes with poisson distribution. Below are the differences which need to be modified from the above case, that is, for all $w \in \{0, 1, ...\}$, $j \in \{1, 2, ...\}$,

$$P\{E(w) = j\} \quad = \quad \lambda^j e^{-\lambda} / j! \tag{3.19}$$

and

$$\begin{aligned} & P\{Q(w) = i_w\} \\ & = \sum_{i_{w-1}=0}^{\infty} P\{Q(w-1) = i_{w-1}\} \\ & (\sum_{k_w=0}^{i_{w-1}} P\{D(w) = k_w / Q(w-1) = i_{w-1}\} \\ & P\{E(w) = i_w - i_{w-1} + k_w\}). \end{aligned} \tag{3.20}$$

IV. CONCLUSION

In this paper, we have analyzed a real existing man-machine system, namely, the satellite tracking/control and brain-satellite dialogue communication system. Certain hitting probability distribution is derived to show some impact of the system. Extensions of the current discussion to the corresponding system with general input traffic pattern and to general network system consisting of multiple regions and multiple base stations are under way. Furthermore, more functionalities and impacts of the brain-satellite system will be addressed in the future paper, which indicate that suitable national and international laws should be developed or applied so that the system can be used suitably and peacefully.

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REFERENCES

[1] F. Ananasso, *Satellite and the Global Information Infrastructure*, Telecommunications, February, 75-81, 1997.

[2] M. Andrews, K. Kumaran, K. Ramanan, A. Stolyar, R. Vijayakumar and P. Whiting, *Providing Quality of Service over a Shared Wireless Link*, IEEE Communication Magazine, Vol.29, No.2, 150-154, 2001.

[3] W. Dai, *Stochastic Adaptive Control for Time-Varying Systems Convergent at the Rate of 1/t*, IFAC Proceedings Series, Vol. 1, No. 8, 179-184, Pergamon Press, U.S.A., 1989.

[4] W. Dai and S. Wang, *Optimal Control Based on a General Exponential Scheduling Rule for a Generalized Switch*, to appear in Proceedings of Communications and Mobile Computing 2009, IEEE Computer Society Press, 2009.

[5] G.C. Goodwin and K.S. Sin, *Adaptive Filtering Prediction and Control*, Prentice-Hall, Englewood Cliffs, NJ, U.S.A., 1984.

[6] K. Johannsen, *Mobile P-Service System Comparison*, International Journal of Satellite Communications, Vol. 13, 453-471, 1995.

[7] M.N. Kay, T. Naselaris, R.J. Prenger and J.L. Gallant, *Identifying Natural Images from Human Brain Activity*, Nature, Vol. 452, 352-355, 2008.

[8] Y. Miyawaki, H. Uchida, O. Yamashita, M. Sato, Y. Morito, H.C. Tanabe, N. Sadato, and Y. Kamitani, *Visual Image Reconstruction from Human Brain Activity using a Combination of Multiscale Local Image Decoders*, Neuron, Vol. 60, issue 5, 915-929, 2008.

[9] J.E. Taylor and K.J. Worsley, *Detecting Sparse Signals in Random Fields, with an Application to Brain Mapping*, Journal of the American Statistical Association 102(479), 913-928, 2007.