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RESEARCH ARTICLE

# Generalized fractional Lévy random fields on Gel'fand triple: A white noise approach

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**Abstract** In this paper, under the first-order moment condition of the infinitely divisible distribution on Gel'fand triple, we use Riesz potential to construct fractional Lévy random fields on Gel'fand triple by white noise approach. We investigate the distribution and sample properties of isotropic and anisotropic fractional Lévy random fields, respectively.

**Keywords** Infinitely divisible distributions, Gel'fand triple, fractional Lévy noise, generalized Lévy random field, fractional generalized Lévy random field, anisotropic Lévy random fields
**MSC** 60E07, 60G20, 60G51, 60G52, 60H40

## 1 Introduction

The fractional Brownian motion (FBM) introduced by Kolmogrov in 1940 and popularized by Mandelbrot and Van Ness in 1968 [13] have been widely applied in mathematical finance and network traffic analysis. A fractional random field is the generalization of the fractional stochastic process in higher dimensions which has been widely used in fluid mechanics, image processing, internet traffic, mathematical finance, etc. (see [2]). Ahn et al. [1] and Ruiz-Medina et al. [16] defined the fractional generalized random fields as fields on fractional Sobolev spaces by Riesz and Bessel potential. Huang et al. [6,7] used white noise approach to investigate the fractional Brownian sheets and fractional stable fields. In [8], Huang et al. constructed the generalized fractional Lévy processes as Lévy white noise functionals. Marquardt [14] investigated the fractional Lévy processes and defined stochastic integration

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for deterministic integrands. As the generalization of fractional Lévy processes in higher dimensions, by white noise approach, Huang et al. [9] constructed the fractional random Lévy random fields and investigated their distribution and sample properties.

Recently, the stochastic evolution equations driven by FBM in Hilbert spaces have received great attentions (cf. [3,4,15,19]). If one considers a scale of Hilbert spaces, the notion of cylindrical processes might be unnecessary. Therefore, it is more natural to consider the fractional processes taking values in the dual space of a countably Hilbertian nuclear space. The most suitable framework should be Gel'fand triple. Wang et al. [21] investigated a class of infinitely divisible distribution on Gel'fand triple by white noise approach. By the Riemann-Liouville fractional integral, Huang et al. [10] defined the fractional Lévy processes and noises on a Gel'fand triple and investigated their distribution properties. Lü et al. [12] further investigated the distribution properties of fractional Lévy processes on a Gel'fand triple restricted to the case that the underlying Lévy processes are centered and square integrable, and defined the stochastic integration with respect to the fractional Lévy processes for deterministic integrands.

In this paper, with a proper restriction on the infinitely divisible distribution on Gel'fand triple, we use Riesz potential to construct fractional Lévy random fields on Gel'fand triple by white noise approach. We investigate the distribution and sample properties of isotropic and anisotropic fractional Lévy random fields, respectively.

The paper is organized as follows. In Section 2, under the first-order moment condition of the infinitely divisible distribution on Gel'fand triple, we construct the tempered generalized Lévy random fields on Gel'fand triple. Based on the generalized Lévy random fields on Gel'fand triple, we use Riesz potential to construct isotropic fractional Lévy random fields in Section 3 and use Riesz poly-potential to construct isotropic fractional Lévy random fields in Section 4, and investigate their distribution properties, respectively.

## 2 Generalized Lévy random fields on Gel'fand triple

Let $E \subset H \subset E^*$ be a real Gel'fand triple generated by $(H, A)$, where $H$ is a real separable Hilbert space with norm $|\cdot|_0$ and inner product $(\cdot,\cdot)$, $A$ is a positive self-adjoint operator in $H$ and $\exists\, \alpha > 0$ such that $A^{-\alpha}$ is nuclear. Define

$$|\cdot|_r := |A^r \cdot|_0, \quad r \in \mathbb{R},$$

and let $E_r$ be the completion of the domain of $A^r$ with respect to $|\cdot|_r$. Then we have a scale of Hilbert spaces $\{E_r, r \in \mathbb{R}\}$, and

$$E := \operatorname*{projlim}_{r>0}\{E_r\}, \quad E^* := \operatorname*{indlim}_{r>0}\{E_{-r}\}.$$

$E$ is a Fréchet nuclear space and $E^*$ is its dual. Denote by $\langle\cdot,\cdot\rangle$ the canonical bilinear form on $E^* \times E$, which is consistent with the inner product of $H$. Moreover, $E$ is densely and continuously embedded in $H$.

For a finite Borel measure $\mu$ on $E^*$, we will denote $\widehat{\mu}$ as its characteristic functional (or Fourier transform), i.e.,

$$\widehat{\mu}(\xi) := \int_{E^*} \mathrm{e}^{\mathrm{i}\langle x,\xi\rangle} \mathrm{d}\mu(x), \quad \xi \in E.$$

A probability measure on $E^*$ is said to be infinitely divisible if for each $n \geqslant 1$, there exists a probability measure $\mu_n$ on $E^*$ such that

$$\widehat{\mu}(\xi) = \widehat{\mu}_n(\xi)^n, \quad \forall\, \xi \in E.$$

We recall a basic result about infinitely divisible distributions (IDD) on $E^*$, for more details, see Huang et al. [10] (for details about IDD on finite dimensional spaces, see [18]).

Let $\mathscr{L}^+(E, E^*)$ be the space of continuous linear operators from $E$ to $E^*$ such that

$$\langle Q\xi, \xi\rangle \geqslant 0, \quad \forall\, \xi \in E,$$

and let $\mathscr{M}_0$ be the set of all Borel measure $\nu$ on $E^*$ with $\nu(\{0\}) = 0$ satisfying that there exists $p > 0$ such that $\nu$ is supported in $E_{-p}$ and

$$\int_{E^*} (|x|_{-p}^2 \wedge 1) \mathrm{d}\nu(x) < \infty. \tag{2.1}$$

**Theorem 2.1** [10] *Let $E \subset H \subset E^*$ be a real Gel'fand triple generated by $(H, A)$ as above. Give $a \in E^*$, $Q \in \mathscr{L}^+(E, E^*)$, and $\nu \in \mathscr{M}_0$. Then there exist $q > 0$ and an IDD $\mu$ on $E^*$ such that*

$$\widehat{\mu}(\xi) = \exp \psi(\xi),$$

*where*

$$\psi(\xi) = \mathrm{i}\langle a, \xi\rangle - \frac{1}{2}\langle Q\xi, \xi\rangle + \int_{E^*} [\mathrm{e}^{\mathrm{i}\langle x,\xi\rangle} - 1 - \mathrm{i}\langle x, \xi\rangle 1_{\{|x|_{-q} \leqslant 1\}}(x)] \mathrm{d}\nu(x), \quad \forall\, \xi \in E. \tag{2.2}$$

We call $(a, Q, \nu)$ the generating triple of $\mu$, and $\nu$ the Lévy measure.

In this paper, we assume that $\mu$ is an IDD on $E^*$ with Lévy measure $\nu$ supported in $E_{-q}$, and

$$\int_{|x|_{-q} \geqslant 1} |x|_{-q} \mathrm{d}\nu(x) < \infty. \tag{2.3}$$

In this case, $\psi(\xi)$ can be written as

$$\psi(\xi) = \mathrm{i}\langle \widetilde{a}, \xi\rangle - \frac{1}{2}\langle Q\xi, \xi\rangle + \int_{E^*} [\mathrm{e}^{\mathrm{i}\langle x,\xi\rangle} - 1 - \mathrm{i}\langle x, \xi\rangle] \mathrm{d}\nu(x), \quad \forall\, \xi \in E, \tag{2.4}$$

where $\widetilde{a}$ is called the mean of $\mu$ satisfying

$$\langle \widetilde{a}, \xi \rangle = \langle a, \xi \rangle + \int_{|x|_{-q} \geqslant 1} \langle x, \xi \rangle \mathrm{d}\nu(x), \quad \forall\, \xi \in E. \tag{2.5}$$

Denote by $\mathscr{S}(\mathbb{R}^d)$ the Schwartz space of rapidly decreasing $C^\infty$-functions on $\mathbb{R}^d$ and by $\mathscr{S}'(\mathbb{R}^d)$ the space of tempered distributions. Then

$$\mathscr{S}(\mathbb{R}^d) \subset L^2(\mathbb{R}^d) \subset \mathscr{S}'(\mathbb{R}^d)$$

is a Gel'fand triple (cf. [5]).

A (tempered) $E^*$-valued generalized random field on probability space $(\Omega, \mathscr{F}, P)$ is a continuous linear map $f \to \dot{X}(f)$ from $\mathscr{S}(\mathbb{R}^d)$ into $E^* \otimes L^0$, where $L^0 = L^0(\Omega, \mathscr{F}, P)$ is the space of random variables on $(\Omega, \mathscr{F}, P)$ equipped with the topology of convergence in probability. Note that $E$ and $L^0$ are Fréchet spaces, and $E$ is nuclear. We have

$$E^* \otimes L^0 \cong \mathscr{L}(E, L^0)$$

(cf. [20]), and

$$\dot{X} \in \mathscr{L}(\mathscr{S}(\mathbb{R}^d), \mathscr{L}(E, L^0)),$$

namely, $(f, \xi) \mapsto \langle \dot{X}(f), \xi \rangle$ is a separately continuous bilinear map of $\mathscr{S}(\mathbb{R}^d) \times E$ into $L^0$, and hence, it is continuous, i.e.,

$$\dot{X} \in \mathscr{B}(\mathscr{S}(\mathbb{R}^d), E; L^0) \cong \mathscr{L}(\mathscr{S} \otimes E, L^0).$$

In other words, $\{\dot{X}(f), f \in \mathscr{S}(\mathbb{R}^d)\}$ is a family of $E^*$-valued R.V. on $(\Omega, \mathscr{F}, P)$ such that

1° $\forall\, a, b \in \mathbb{R}^d,\ f, g \in \mathscr{S}(\mathbb{R}^d),\ \dot{X}(af + bg) = a\dot{X}(f) + b\dot{X}(g)$ a.s.;

2° $f_n \to f$ in $\mathscr{S}(\mathbb{R}^d)$ implies that $\forall\, \xi \in E$, $\langle \dot{X}(f_n), \xi \rangle \to \langle \dot{X}(f), \xi \rangle$ in probability. (By linearity, one may replace convergence in probability by convergence in law and by Itô regularization lemma [11], it has a version which is $\mathscr{S}' \otimes E^*$-valued R.V.)

If, moreover,

3° $f, g \in \mathscr{S}(\mathbb{R}^d),\ fg = 0$ implies that $\dot{X}(f)$ and $\dot{X}(g)$ are independent,

then $\{\dot{X}(f), f \in \mathscr{S}(\mathbb{R}^d)\}$ is called an $E^*$-valued (tempered) white noise.

**Theorem 2.2** *Assume that $\mu$ is an IDD on $E^*$ with characteristic exponent given by* (2.2) *and Lévy measure $\nu$ satisfying* (2.3). *Then there exists a tempered $E^*$-valued white noise $\{\dot{X}(f), f \in \mathscr{S}(\mathbb{R}^d)\}$ on some probability space $(\Omega, \mathscr{F}, P)$ such that*

$$\mathbb{E}[\mathrm{e}^{\mathrm{i}\langle \dot{X}(f), \xi \rangle}] = \exp\left\{ \int_{\mathbb{R}^d} \psi(f(\overline{s})\xi) \mathrm{d}\overline{s} \right\}, \quad \xi \in E, \tag{2.6}$$

*where $\psi$ is given by* (2.2) (*or equivalently,* (2.4)), $\overline{s} = (s_1, \ldots, s_d) \in \mathbb{R}^d$. *This expression extends to $f \in L^1(\mathbb{R}^d) \cap L^2(\mathbb{R}^d)$, and*

$$X(\overline{t}) := \dot{X}(1_{[0,\overline{t}]}), \quad \overline{t} = (t_1, \ldots, t_d), \quad t_i \geqslant 0, \quad i = 1, 2, \ldots, d, \tag{2.7}$$

*is an $E^*$-valued Lévy random field.*

*Especially, for $d=1$, $X=\{X(t),t\geqslant 0\}$ is an $E^*$-valued Lévy process.*

*Proof* Take $q>0$ such that $a\in E_{-q}$, $Q\in\mathscr{L}(E_q,E_{-q})$, and $\nu$ is supported in $E_{-q}$ with

$$\int_{|x|_{-q}>1}|x|_{-q}\mathrm{d}\nu(x)<\infty.$$

Then

$$\begin{aligned}|\psi(\xi)|&\leqslant |a|_{-q}\,|\xi|_q+\frac{1}{2}\,\|Q\|_{L(E_q,E_{-q})}\,|\xi|_q^2\\&\quad+|\xi|_q\int_{|x|_{-q}>1}|x|_{-q}\mathrm{d}\nu(x)+\frac{1}{2}\,|\xi|_q^2\int_{|x|_{-q}\leqslant 1}|x|_{-q}^2\mathrm{d}\nu(x)\\&=c_q^{(1)}|\xi|_q+c_q^{(2)}|\xi|_q^2,\end{aligned}$$

where $c_q^{(1)},c_q^{(2)}\geqslant 0$ are constants. Hence, for $f\in L^1(R^d)\cap L^2(R^d),\ \xi\in E,$

$$\int_{\mathbb{R}^d}|\psi(f(\overline{s})\xi)|\mathrm{d}\overline{s}\leqslant c_q^{(1)}|\xi|_q\,\|f\|_{L^1}+c_q^{(2)}|\xi|_q^2\,\|f\|_{L^2}^2. \tag{2.8}$$

Now, the right-hand side of (2.6) defines a positive-definite continuous functionals on $\mathscr{S}(\mathbb{R}^d)\otimes E$. By the Minlos theorem, there exists a probability measure $\mathbb{P}$ on $\Omega\equiv\mathscr{S}'(\mathbb{R}^d)\otimes E^*$ such that

$$\int_\Omega \mathrm{e}^{\mathrm{i}\langle\omega,f\otimes\xi\rangle}\mathrm{d}\mathbb{P}(\omega)=\exp\left\{\int_{\mathbb{R}^d}\psi(f(\overline{s})\xi)\mathrm{d}\overline{s}\right\}. \tag{2.9}$$

Define $\dot{X}(f,\omega)\in E^*$ via $\langle\dot{X}(f,\omega),\xi\rangle=\langle\omega,f\otimes\xi\rangle$. Then $\{\dot{X}(f,\omega),f\in\mathscr{S}(\mathbb{R}^d)\}$ is a tempered $E^*$-valued white noise on $(\Omega,\mathscr{F},P)$.

In view of (2.8), it has continuous extension to $f\in L^1(\mathbb{R}^d)\cap L^2(\mathbb{R}^d)$. Putting $f(s)=1_{[0,\overline{t}]}(\overline{s})$, we obtain an $E^*$-valued Lévy random fields $X=\{X(\overline{t}),\ \overline{t}\in\mathbb{R}_+^d\}$ with

$$\mathbb{E}[\mathrm{e}^{\mathrm{i}\langle X(\overline{t}),\xi\rangle}]=\mathrm{e}^{\mathrm{Leb}([0,\overline{t}])\psi(\xi)}=\mathrm{e}^{(\prod_{j=1}^d t_j)\psi(\xi)},$$

where Leb is the Lebesgue measure on $\mathbb{R}^d$. □

**Remark** 1° For $S\in\mathscr{B}(\mathbb{R}^d)$, define $X(S)$ by

$$\mathbb{E}[\mathrm{e}^{\mathrm{i}\langle X(S),\xi\rangle}]=\mathrm{e}^{\mathrm{Leb}(S)\psi(\xi)}. \tag{2.10}$$

Then $\{X(S),S\in\mathscr{B}(\mathbb{R}^d)\}$ is an $E^*$-valued Lévy random measure on $\mathbb{R}^d$. Formally, we write

$$\dot{X}(f)=\int_{\mathbb{R}^d}f(\overline{s})\mathrm{d}X(\overline{s}),\quad f\in L^1(\mathbb{R}^d)\cap L^2(\mathbb{R}^d).$$

2° Extending Proposition 5.1 and Corollary 5.2 of [12] to the $\mathbb{R}^d$ case, we deduce that for $f\in L^1(\mathbb{R}^d)\cap L^2(\mathbb{R}^d),\ \forall\,\xi\in E,\ \langle\dot{X}(f),\xi\rangle$ is infinitely divisible.

$3^\circ$ If $\mu$ is strictly $\alpha$-stable ($1 < \alpha < 2$) on $E^*$, then by [10, Theorem 3.2], there exists a constant $C > 0$ such that

$$|\psi(\xi)| \leqslant C|\xi|_q^\alpha, \quad \forall\, \xi \in E.$$

Therefore,

$$\int_{\mathbb{R}^d} |\psi(f(\overline{s})\xi)| \mathrm{d}\overline{s} \leqslant C|\xi|_q^\alpha \, \|f\|_{L^\alpha}^\alpha. \tag{2.11}$$

It follows that for $f \in L^\alpha(\mathbb{R}^d)$, (2.6) holds and (2.7) defines an $E^*$-valued $\alpha$-stable Lévy random field.

## 3 Isotropic generalized fractional Lévy random fields on Gel'fand triple

In this section, we use the Riesz potential to define the generalized isotropic fractional Lévy random fields on Gel'fand triple $E \subset H \subset E^*$. For $\beta \in (0, d)$, $f \in \mathscr{S}(\mathbb{R}^d)$, the Riesz potential is defined by

$$I^\beta f(\overline{x}) := c_\beta \int_{\mathbb{R}^d} \frac{f(\overline{y})}{|\overline{x} - \overline{y}|^{d-\beta}} \, \mathrm{d}\overline{y}, \tag{3.1}$$

where

$$c_\beta = \pi^{d/2} 2^\beta \Gamma\Big(\frac{\beta}{2}\Big) \Big/ \Gamma\Big(\frac{d-\beta}{2}\Big)$$

is the Tauberian constant.

From the definition, it is easy to check that $I^\beta$ satisfies the following properties.

(i) For $\alpha, \beta \in (0, d)$ and $\alpha + \beta < d$,

$$I^\alpha I^\beta = I^{\alpha+\beta}. \tag{3.2}$$

(ii) For translation operator $\tau_{\overline{h}} f(\overline{x}) := f(\overline{x} - \overline{h}) := f(x_1 - h_1, \ldots, x_d - h_d)$, $\overline{h} = (h_1, \ldots, h_d)$,

$$\tau_{\overline{h}} I^\beta = I^\beta \tau_{\overline{h}}. \tag{3.3}$$

(iii) For dilation operator $\Pi_\lambda f(\overline{x}) := f(\lambda \cdot \overline{x})$ with $\lambda > 0$,

$$\Pi_\lambda I^\beta = \lambda^\beta I^\beta \Pi_\lambda. \tag{3.4}$$

**Proposition 3.1** [17] (1) *For $0 < \beta < d$, $1 \leqslant p < d/\beta$, $\forall\, f \in L^p(\mathbb{R}^d)$, the integral in* (3.1) *converges a.e.*

(2) *$I^\beta$ are bounded from $L^p(\mathbb{R}^d)$ to $L^q(\mathbb{R}^d)$ if and only if $1 < p < d/\beta$ and $\frac{1}{q} = \frac{1}{p} - \frac{\beta}{d}$.*

**Proposition 3.2** [9] *For $0 < \beta < \min(d/2, 1)$, the operator $I^\beta \colon \mathscr{S}(\mathbb{R}^d) \to L^1(\mathbb{R}^d) \cap L^2(\mathbb{R}^d)$ is continuous.*

Take $\Omega \equiv \mathscr{S}'(\mathbb{R}^d) \otimes E^*$ and $\mathbb{P}$ defined by (2.9). Since for $0 < \beta < \min(d/2, 1)$, the operator $I^\beta\colon \mathscr{S}(\mathbb{R}^d) \to L^1(\mathbb{R}^d) \cap L^2(\mathbb{R}^d)$ is continuous, it defines a probability measure $\mathbb{P}_\beta$ on $\Omega$ by

$$\int_\Omega \mathrm{e}^{\mathrm{i}\langle\omega, f\otimes\xi\rangle}\mathrm{d}\mathbb{P}_\beta(\omega) = \exp\bigg\{\int_{\mathbb{R}^d} \psi(I^\beta f(\overline{s})\xi)\mathrm{d}\overline{s}\bigg\}. \tag{3.5}$$

By the Itô regularization lemma, we obtain a measurable map $T_\beta\colon \Omega \to \Omega$ such that

$$\langle T_\beta\omega, f\otimes\xi\rangle = \langle\omega, I^\beta f\otimes\xi\rangle, \quad \mathbb{P}\text{-a.s.}$$

Therefore, $\mathbb{P}_\beta = \mathbb{P}\circ T_\beta^{-1}$ is the image measure of $\mathbb{P}$ induced by the map $T_\beta$.

**Theorem 3.3** *Let $\{\dot{X}(f), f\in\mathscr{S}(\mathbb{R}^d)\}$ be a tempered $E^*$-valued white noise defined in Theorem* 2.2. *Then, for $0<\beta<\min(d/2,1)$,*

$$\dot{X}^\beta(f) := \dot{X}(I^\beta f), \quad f\in\mathscr{S}(\mathbb{R}^d), \tag{3.6}$$

*is a tempered $E^*$-valued generalized random field. We refer it as the generalized fractional Lévy random field.*

*Proof* By (2.6), we have

$$\mathbb{E}[\mathrm{e}^{\mathrm{i}\langle\dot{X}^\beta(f),\xi\rangle}] = \exp\bigg\{\int_{\mathbb{R}^d}\psi(I^\beta f(\overline{s})\xi)\mathrm{d}\overline{s}\bigg\}. \tag{3.7}$$

Using estimation (2.8), we get

$$\int_{\mathbb{R}^d}|\psi(I^\beta f(\overline{s})\xi)|\mathrm{d}\overline{s} \leqslant c_q^{(1)}|\xi|_q\,\|I^\beta f\|_{L^1} + c_q^{(2)}|\xi|_q^2\,\|I^\beta f\|_{L^2}^2.$$

Since $I^\beta\colon \mathscr{S}(\mathbb{R}^d)\to L^1(\mathbb{R}^d)\cap L^2(\mathbb{R}^d)$ is continuous, $f\mapsto\dot{X}^\beta(f)$ is a continuous map from $\mathscr{S}(\mathbb{R}^d)$ into $E^*\otimes L^0(\Omega,\mathscr{F},P)$. □

**Proposition 3.4** *Suppose that $0<\beta<d/2$, $\nu$ is supported in $E_{-q}$ satisfying*

$$\int_{|x|_{-q}\geqslant 1}|x|_{-q}^2\mathrm{d}\nu(x)<\infty$$

*(denoted by $\nu\in\mathscr{M}_2$), $\widetilde{a}=0$, and the characteristic exponent of $\mu$ is given by*

$$\psi(\xi) = -\frac{1}{2}\langle Q\xi,\xi\rangle + \int_{E^*}[\mathrm{e}^{\mathrm{i}\langle x,\xi\rangle} - 1 - \mathrm{i}\langle x,\xi\rangle]\mathrm{d}\nu(x). \tag{3.8}$$

*Let $\{\dot{X}^\beta(f),\, f\in\mathscr{S}(\mathbb{R}^d)\}$ be an $E^*$-valued random field defined in Theorem* 3.3. *Then $\dot{X}^\beta(f)$ is an IDD on $E^*$ with characteristic exponent*

$$\psi_\beta^f(\xi) = \int_{\mathbb{R}^d}\psi(I^\beta f(\overline{s})\xi)\mathrm{d}\overline{s}$$

$$= \mathrm{i}\langle \widetilde{a}_\beta^f, \xi\rangle - \frac{1}{2}\langle Q_\beta^f \xi, \xi\rangle + \int_{E^*} [\mathrm{e}^{\mathrm{i}\langle x,\xi\rangle} - 1 - \mathrm{i}\langle x, \xi\rangle] \mathrm{d}\nu_\beta^f(x), \quad \forall\, \xi \in E, \tag{3.9}$$

*where*

$$\widetilde{a}_\beta^f = \widetilde{a}\int_{\mathbb{R}^d} (I^\beta f)(\overline{s})\mathrm{d}\overline{s} = 0, \quad Q_\beta^f = \|I^\beta f\|_{L^2}^2 Q,$$

$$\nu_\beta^f(B) = \int_{\mathbb{R}^d}\int_{E^*} 1_B((I^\beta f)(\overline{s})x)\mathrm{d}\nu(x)\mathrm{d}\overline{s}, \quad \forall\, B \in \mathscr{B}(E^*).$$

*Proof* (3.9) is directly followed by (2.4) and (3.7). Since $f \in \mathscr{S}(\mathbb{R}^d)$, it is easy to see that $a_\beta^f \in E^*$, $Q_\beta^f \in \mathscr{L}^+(E, E^*)$. Thus, it suffices to verify that $\nu_\beta^f$ is a Lévy measure on $E^*$. Since $\nu$ is a Lévy measure supported in $E_{-q}$, and $f \in \mathscr{S}(\mathbb{R}^d)$, we know that $\nu_\beta^f$ is supported in $E_{-q}$, and

$$\begin{aligned}\int_{E^*} (|x|_{-q}^2 \wedge 1)\mathrm{d}\nu_\beta^f(x) &\leqslant \int_{E^*} |x|_{-q}^2 \mathrm{d}\nu_\beta^f(x) \\ &= \int_{\mathbb{R}^d}\int_{E^*} |(I^\beta f)(\overline{s})x|_{-q}^2 \mathrm{d}\nu(x)\mathrm{d}\overline{s} \\ &= \|I^\beta f\|_{L^2}^2 \int_{E^*} |x|_{-q}^2 \mathrm{d}\nu(x) \\ &< \infty,\end{aligned}$$

that is, $\nu_\beta^f$ is a Lévy measure on $E^*$. Hence, $\dot{X}^\beta(f)$ is an IDD on $E^*$. □

**Corollary 3.5** *If* $0 < \beta < d/2$, $\nu \in \mathscr{M}_2$, $\widetilde{a} = 0$, *and the characteristic exponent of* $\mu$ *is given by* (3.8), *then for any* $f \in \mathscr{S}(\mathbb{R}^d)$, $\xi, \eta \in E$, *we have*

$$\mathbb{E}[\langle \dot{X}^\beta(f), \xi\rangle] = 0,$$

$$\begin{aligned}\mathbb{E}[\langle \dot{X}^\beta(f), \xi\rangle\langle \dot{X}^\beta(g), \eta\rangle] &= (I^\beta f, I^\beta g)_{L^2}\left\{\langle Q\xi, \eta\rangle + \int_{E^*} \langle x, \xi\rangle\langle x, \eta\rangle \mathrm{d}\nu(x)\right\} \\ &= \left\{\langle Q\xi, \eta\rangle + \int_{E^*} \langle x, \xi\rangle\langle x, \eta\rangle \mathrm{d}\nu(x)\right\} \\ &\quad \times \int_{\mathbb{R}^d} |\overline{t}|^{-2\beta} \widehat{f}(\overline{t})\widehat{g}(\overline{t})\mathrm{d}\overline{t},\end{aligned}$$

*where* $\widehat{f}$ *is the Fourier transform of* $f$.

*Proof* By Proposition 3.4, $\widetilde{a}_\beta^f = 0$, and hence,

$$\mathbb{E}[\langle \dot{X}^\beta(f), \xi\rangle] = 0, \quad \forall\, \xi \in E.$$

From the proof of Proposition 3.4, $\nu_\beta^f \in \mathscr{M}_2$, and by (3.7), we have

$$\mathbb{E}[\langle \dot{X}^\beta(f), \xi\rangle^2] = -\frac{\mathrm{d}^2}{\mathrm{d}u^2}\mathbb{E}[\mathrm{e}^{\mathrm{i}u\langle \dot{X}^\beta(f),\xi\rangle}]|_{u=0}$$

$$
\begin{aligned}
&= -\frac{\mathrm{d}^2}{\mathrm{d}u^2}\left\{\exp\left\{\int_{\mathbb{R}^d}\psi(I^\beta f(\overline{s})u\xi)\mathrm{d}\overline{s}\right\}\right\}\Big|_{u=0}\\
&= \|I^\beta f\|_{L^2}\left\{\langle Q\xi,\xi\rangle+\int_{E^*}\langle x,\xi\rangle^2\mathrm{d}\nu(x)\right\}\\
&= \left\{\langle Q\xi,\xi\rangle+\int_{E^*}\langle x,\xi\rangle^2\mathrm{d}\nu(x)\right\}\int_{\mathbb{R}^d}|\overline{t}|^{-2\beta}\widehat{f}(\overline{t})^2\mathrm{d}\overline{t}.
\end{aligned}
$$

By polarization, we get the desired result. □

**Definition 3.6** Let $\{\dot{X}(f), f\in\mathscr{S}(\mathbb{R}^d)\}$ be a tempered generalized $E^*$-valued random field defined in Theorem 2.2. Then

$$
X^\beta(\overline{t}) := \dot{X}(I^\beta 1_{[0,\overline{t}]}),\quad \overline{t}=(t_1,\ldots,t_d),\quad t_k\geqslant 0,\quad k=1,2,\ldots,d,
$$

is called $\beta$-fractional Lévy random field whenever the right-hand side makes sense.

Note that $I^\beta 1_{[0,t]}\in L^2(\mathbb{R}^d)$ but $I^\beta 1_{[0,t]}\overline{\in} L^1(\mathbb{R}^d)$. Then $X^\beta(t)$ does not make sense in general. However, it is well defined in the following two important cases.

**Case 1** $0<\beta<d/2,\ \nu\in\mathscr{M}_2,\ \widetilde{a}=0$, and the characteristic exponent of $\mu$ is given by

$$
\psi(\xi) = -\frac{1}{2}\langle Q\xi,\xi\rangle+\int_{E^*}[\mathrm{e}^{\mathrm{i}\langle x,\xi\rangle}-1-\mathrm{i}\langle x,\xi\rangle]\mathrm{d}\nu(x).
$$

Therefore, $\exists\, q>0,\ c_q>0$ such that

$$
\int_{\mathbb{R}^d}|\psi(f(\overline{s})\xi)|\mathrm{d}\overline{s}\leqslant c_q|\xi|_q^2\,\|f\|_{L^2}^2.
$$

Taking $p=2d/(d+2\beta)$, since $I^\beta\in\mathscr{L}(L^p(\mathbb{R}^d),L^2(\mathbb{R}^d))$, we have $I^\beta 1_{[0,\overline{t}]}\in L^2(\mathbb{R}^d)$.

**Case 2** $1<\alpha<2,\ 0<\beta<(1-\frac{1}{\alpha})d,\ \mu$ is $\alpha$-stable. By (2.11), we have

$$
\int_{\mathbb{R}^d}|\psi(f(\overline{s})\xi)|\mathrm{d}\overline{s}\leqslant C|\xi|_q^\alpha\,\|f\|_{L^\alpha}^\alpha.
$$

Taking $p=d\alpha/(d+\alpha\beta)\in(1,2)$, $I^\beta\in\mathscr{L}(L^p(\mathbb{R}^d),L^\alpha(\mathbb{R}^d))$, we have $I^\beta 1_{[0,\overline{t}]}\in L^\alpha(\mathbb{R}^d)$.

Now, we investigate the distribution and sample properties of fractional generalized random field by regarding it as a functional of tempered generalized white noise.

**Proposition 3.7** *The $E^*$-valued generalized fractional Lévy random field $\{\dot{X}^\beta(f), f\in\mathscr{S}(\mathbb{R}^d)\}$ is stationary, i.e., $\forall\ \overline{h}\in\mathbb{R}^d,\ \forall\ f\in\mathscr{S}(\mathbb{R}^d),\ \dot{X}^\beta(f)=^d\dot{X}^\beta(\tau_{\overline{h}}f)$.*

*Proof* Since for translation operator $\tau_h$, we have $\tau_{\overline{h}} I^\beta = I^\beta \tau_{\overline{h}}$. It follows from (3.7) and the invariance of Lebesgue measure by translation that

$$\begin{aligned}\mathbb{E}[\exp\{i\langle X^\beta(\tau_{\overline{h}} f), \xi\rangle\}] &= \exp\left\{\int_{\mathbb{R}^d} \psi(I^\beta \tau_{\overline{h}} f(\overline{s})\xi)\mathrm{d}\overline{s}\right\} \\ &= \exp\left\{\int_{\mathbb{R}^d} \psi(\tau_{\overline{h}} I^\beta f(\overline{s})\xi)\mathrm{d}\overline{s}\right\} \\ &= \exp\left\{\int_{\mathbb{R}^d} \psi(I^\beta f(\overline{s}-\overline{h})\xi)\mathrm{d}\overline{s}\right\} \\ &= \exp\left\{\int_{\mathbb{R}^d} \psi(I^\beta f(\overline{s})\xi)\mathrm{d}\overline{s}\right\} \\ &= \mathbb{E}[\exp\{i\langle \dot{X}^\beta(f), \xi\rangle\}],\end{aligned}$$

which means that

$$\dot{X}^\beta(f) =^d \dot{X}^\beta(\tau_h f), \quad \forall\ \overline{h} \geqslant 0,\ \forall\ f \in \mathscr{S}(\mathbb{R}^d). \qquad \square$$

When the fractional random field $\{X^\beta(\overline{t}), \overline{t} \in \mathbb{R}^d_+\}$ is well defined, the stationariness of its increments can be directly deduced by Proposition 3.7.

**Proposition 3.8** *The $E^*$-valued generalized fractional Lévy random field $\{\dot{X}^\beta(f), f \in \mathscr{S}(\mathbb{R}^d)\}$ is isotropic, i.e., for all rotations and reflections operator $A$ on the Euclidean space $\mathbb{R}^d$,*

$$\dot{X}^\beta(f) =^d \dot{X}^\beta(Af), \quad f \in \mathscr{S}(\mathbb{R}^d).$$

*Proof* We check that, for all rotations and reflections operator $A$ on the Euclidean space $\mathbb{R}^d$, $AI^\beta = I^\beta A$. Since the Lebesgue measure is invariant under rotations and reflections, we can easily get the assertion by the similar proof of Proposition 3.7. $\square$

**Proposition 3.9** *Suppose that $1 < \alpha < 2$, $0 < \beta < (1 - \frac{1}{\alpha})d$, and $\mu$ is $\alpha$-stable. Then the $E^*$-valued generalized $\beta$-fractional $\alpha$-stable Lévy random field $\dot{X}^\beta_\alpha = \{X^\beta_\alpha(f), f \in \mathscr{S}(\mathbb{R}^d)\}$ is self-similar with Hurst parameter $H = \beta + \frac{d}{\alpha}$, i.e.,*

$$\lambda^H \dot{X}^\beta_\alpha(\Pi_\lambda f) =^d \dot{X}^\beta_\alpha(f).$$

*On the other hand, if the $E^*$-valued generalized $\beta$-fractional random field $X^\beta = \{X^\beta(t),\ t \geqslant 0\}$ is $H$- self-similar, then $X$ is $\frac{H-\beta}{d}$-stable.*

*Proof* For dilation operator $\lambda > 0$, we have

$$\Pi_\lambda I^\beta = \lambda^\beta I^\beta \Pi_\lambda.$$

Then to prove

$$\lambda^{\beta + \frac{d}{\alpha}} \dot{X}^\beta_\alpha(\Pi_\lambda f) =^d \dot{X}^\beta_\alpha(f),$$

it suffices to observe that $\forall\ \xi \in E$,

$$
\begin{aligned}
\mathbb{E}[\exp\{\mathrm{i}\langle\lambda^{\beta+\frac{d}{\alpha}}\dot{X}_\alpha^\beta(\Pi_\lambda f),\xi\rangle\}] &= \exp\left\{\int_{\mathbb{R}^d}\psi(\lambda^{\beta+\frac{d}{\alpha}}I^\beta\Pi_\lambda f(\overline{s})\xi)\mathrm{d}\overline{s}\right\}\\
&= \exp\left\{\int_{\mathbb{R}^d}\psi(\lambda^{d/\alpha}\Pi_\lambda I^\beta f(\overline{s})\xi)\mathrm{d}\overline{s}\right\}\\
&= \exp\left\{\int_{\mathbb{R}^d}\lambda^d\psi(I^\beta f(\lambda\overline{s})\xi)\mathrm{d}\overline{s}\right\}\\
&= \exp\left\{\int_{\mathbb{R}^d}\psi(I^\beta f(\overline{s})\xi)\mathrm{d}\overline{s}\right\}\\
&= \mathbb{E}[\exp\{\mathrm{i}\langle\dot{X}_\alpha^\beta(f),\xi\rangle\}].
\end{aligned}
$$

Similarly, if $X^\beta$ is self-similar with Hurst parameter $H$, then $\forall\ \lambda > 0$, $\lambda\psi(\xi) = \psi(\lambda^{H-\beta}\xi)$, which means that $X$ is $\frac{d}{H-\beta}$-stable. □

In general, the generalized $\beta$-fractional Lévy random field $\dot{X}^\beta = \{X^\beta(f), f \in \mathscr{S}(\mathbb{R}^d)\}$ is self-similar if and only if $X$ is stable.

## 4 Anisotropic generalized fractional Lévy random fields on Gel'fand triple

In this section, we extend the isotropic generalized fractional Lévy random fields to the anisotropic case by virtue of Riesz poly-potential.

Let

$$
\overline{\beta} = (\beta_1,\dots,\beta_d),\quad 0<\beta_k<\frac{1}{2},\quad k=1,2,\dots,d,
$$

$$
f\in\mathscr{S}(\mathbb{R}^d),\quad \gamma_d(\overline{\beta}) = 2^d\prod_{k=1}^d\Gamma(\beta_k)\cos\frac{\beta_k\pi}{2}.
$$

The fractional power $(-\Delta)^{-\overline{\beta}/2}$ of the Laplace operator is known as the Riesz poly-potential:

$$
I^{\overline{\beta}}f(\overline{t}) := \frac{1}{\gamma_d(\overline{\beta})}\int_{\mathbb{R}^d}\frac{f(\overline{s})\mathrm{d}\overline{s}}{|\overline{t}-\overline{s}|^{1-\overline{\beta}}},\quad f\in\mathscr{S}(\mathbb{R}^d), \tag{4.1}
$$

where

$$
|\overline{t}-\overline{s}|^{1-\overline{\beta}} = \prod_{k=1}^d|t_k-s_k|^{1-\beta_k},\quad \overline{t}=(t_1,\dots,t_d),\quad \overline{s}=(s_1,\dots,s_d).
$$

**Theorem 4.1** [17] *The poly-potential operator $I^{\overline{\beta}}$ is bounded from $L^{\overline{p}}$ to $L^{\overline{q}}$ with $\overline{p}=(p_1,\dots,p_d)$, $\overline{q}=(q_1,\dots,q_d)$ if and only if*

$$
1<p_k<\frac{1}{\beta_k},\quad q_k=\frac{p_k}{1-\beta_kp_k},\quad k=1,2,\dots,d, \tag{4.2}
$$

*where $L^{\overline{q}}$ is the Banach space of functions with mixed norm*

$$\|f\|_{\overline{p}} = \left\{ \int_{\mathbb{R}} \left\{ \cdots \left\{ \int_{\mathbb{R}} \left[ \int_{\mathbb{R}} |f(s_1, \ldots, s_d)|^{p_1} \mathrm{d}s_1 \right]^{\frac{p_2}{p_1}} \mathrm{d}s_2 \right\}^{\frac{p_3}{p_2}} \cdots \right\}^{\frac{p_d}{p_{d-1}}} \mathrm{d}s_d \right\}^{\frac{1}{p_d}} < \infty. \tag{4.3}$$

The following property related to Riesz poly-potential plays a key role in the construction of anisotropic generalized fractional Lévy random fields.

**Proposition 4.2** [9] *For $\overline{\beta} = (\beta_1, \ldots, \beta_d),\ 0 < \beta_k < 1/2,\ k = 1, \ldots, d$, the operator $I^{\overline{\beta}}\colon\ \mathscr{S}(\mathbb{R}^d) \to L^1(\mathbb{R}^d) \cap L^2(\mathbb{R}^d)$ is continuous.*

The construction of anisotropic generalized fractional Lévy random fields is similar to the isotropic case. Take $\Omega \equiv \mathscr{S}'(\mathbb{R}^d) \otimes E^*$, and $\mathbb{P}$ is defined by (2.9). Since for $\overline{\beta} = (\beta_1, \ldots, \beta_d),\ 0 < \beta_k < 1/2,\ k = 1, \ldots, d$, the operator $I^{\overline{\beta}}\colon\ \mathscr{S}(\mathbb{R}^d) \to L^1(\mathbb{R}^d) \cap L^2(\mathbb{R}^d)$ is continuous, it defines a probability measure $\mathbb{P}_{\overline{\beta}}$ on $\Omega$ by

$$\int_{\Omega} \mathrm{e}^{\mathrm{i}\langle \omega, f\otimes\xi\rangle} \mathrm{d}\mathbb{P}_{\overline{\beta}}(\omega) = \exp\left\{ \int_{\mathbb{R}^d} \psi(I^{\overline{\beta}} f(\overline{s})\xi) \mathrm{d}\overline{s} \right\}. \tag{4.4}$$

By the Itô regularization lemma, we obtain a measurable map $T_{\overline{\beta}}\colon\ \Omega \to \Omega$ such that

$$\langle T_{\overline{\beta}}\omega, f \otimes \xi\rangle = \langle \omega, I^{\overline{\beta}} f \otimes \xi\rangle, \quad \mathbb{P}\text{-a.s.}$$

Therefore, $\mathbb{P}_{\overline{\beta}} = \mathbb{P} \circ T_{\overline{\beta}}^{-1}$ is the image measure of $\mathbb{P}$ induced by the map $T_{\overline{\beta}}$.

**Theorem 4.3** *Let $\{\dot{X}(f), f \in \mathscr{S}(\mathbb{R}^d)\}$ be a tempered $E^*$-valued white noise defined in Theorem 2.2. Then, for $\overline{\beta} = (\beta_1, \ldots, \beta_d),\ 0 < \beta_k < 1/2,\ k = 1, \ldots, d$,*

$$\dot{X}^{\overline{\beta}}(f) := \dot{X}(I^{\overline{\beta}} f), \quad f \in \mathscr{S}(\mathbb{R}^d), \tag{4.5}$$

*is a tempered $E^*$-valued generalized random field. We refer it as the generalized anisotropic fractional Lévy random field.*

*Proof* Based on Proposition 4.2, the proof is similar to that of Theorem 3.3, we omit it here. □

The real-valued anisotropic Brownian sheets and stable fields have been studied by Huang et al. [6,7], and the real-valued anisotropic Lévy random field have been studied by Huang and Li [9], all of these processes are not Euclidean invariant.

Suppose that $\mu$ is $\alpha$-stable, $1 < \alpha < 2,\ 0 < \beta_k < 1 - \frac{1}{\alpha},\ k = 1, 2, \ldots, d$. By (2.11), we have

$$\int_{\mathbb{R}^d} |\psi(f(\overline{s})\xi)| \mathrm{d}\overline{s} \leqslant C|\xi|_q^{\alpha}\, \|f\|_{L^{\alpha}}^{\alpha}.$$

Taking

$$p_k = \frac{\alpha}{1 + \alpha\beta_k} \in \left(1, \frac{1}{\beta_k}\right), \quad I^{\overline{\beta}} \in \mathscr{L}(L^{\overline{p}}(\mathbb{R}^d), L^{\alpha}(\mathbb{R}^d)),$$

we have $I^{\overline{\beta}}1_{[0,\overline{t}]} \in L^\alpha(\mathbb{R}^d)$. Hence, the anisotropic $\alpha$-stable $\overline{\beta}$ fractional Lévy random field

$$X_\alpha^{\overline{\beta}}(\overline{t}) := \dot{X}(I^{\overline{\beta}}1_{[0,\overline{t}]}), \quad \overline{t} \in \mathbb{R}^d, \tag{4.6}$$

is well defined.

If we define the increments as

$$\begin{aligned}\Delta_{[\overline{s},\overline{t}]}X = X(t_1,\dots,t_d) - \sum_{i=1}^d X(t_1,\dots,s_i,\dots,t_d) \\ + \sum_{i<j} X(t_1,\dots,s_i,\dots,s_j,\dots,t_d) + \cdots + (-1)^d X(s_1,\dots,s_d),\end{aligned} \tag{4.7}$$

then $X_\alpha^{\overline{\beta}}$ has stationary increments.

**Proposition 4.4** *Suppose that $1 < \alpha < 2$, $0 < \beta_k < 1 - \frac{1}{\alpha}$, $k = 1, 2, \dots, d$, $\mu$ is $\alpha$-stable, $X_\alpha^{\overline{\beta}}$ is the anisotropic $\alpha$-stable $\overline{\beta}$ fractional Lévy random field defined by $\mu$. Then $X_\alpha^{\overline{\beta}}$ is self-similar with Hurst parameter*

$$H = \Big(\beta_1 + \frac{1}{\alpha}, \dots, \beta_d + \frac{1}{\alpha}\Big),$$

*that is, for any $\overline{a} = (a_1, \dots, a_d) \in \mathbb{R}_+^d$,*

$$X_\alpha^{\overline{\beta}}(a \circ \overline{t}) =^d a^H X_\alpha^{\overline{\beta}}(t),$$

*where*

$$a \circ \overline{t} := (a_1 t_1, \dots, a_d t_d), \quad a^H := \prod_{j=1}^d a_j^{H_j}.$$

*Proof* For any $\overline{a} = (a_1, \dots, a_d) \in \mathbb{R}_+^d$, we have

$$I^{\overline{\beta}}(\overline{a} \circ f) = \overline{a}^{-\overline{\beta}}(\overline{a} \circ I^{\overline{\beta}} f),$$

where

$$(\overline{a} \circ f)(\overline{t}) := f(a_1 t_1, \dots, a_d t_d).$$

Then to prove

$$X_\alpha^{\overline{\beta}}(a \circ \overline{t}) =^d a^H X_\alpha^{\overline{\beta}}(t),$$

taking $f = 1_{[0,\overline{t}]}$, it suffices to observe that $\forall\ \xi \in E$,

$$\begin{aligned}\mathbb{E}[\exp\{\mathrm{i}\langle a^H \dot{X}_\alpha^{\overline{\beta}}(a \circ f), \xi\rangle\}] &= \exp\Big\{\int_{\mathbb{R}^d} \psi(a^H I^{\overline{\beta}}(a \circ f)(\overline{s})\xi)\mathrm{d}\overline{s}\Big\} \\ &= \exp\Big\{\int_{\mathbb{R}^d} \psi(\overline{a}^{H-\overline{\beta}}(I^{\overline{\beta}} f)(\overline{a} \circ \overline{s})\xi)\mathrm{d}\overline{s}\Big\} \\ &= \exp\Big\{\int_{\mathbb{R}^d} \Big(\prod_{j=1}^d a_j\Big)\psi((I^{\overline{\beta}} f)(\overline{a} \circ \overline{s})\xi)\mathrm{d}\overline{s}\Big\}\end{aligned}$$

$$= \exp\left\{\int_{\mathbb{R}^d} \psi(I^{\overline{\beta}} f(\overline{s})\xi)\mathrm{d}\overline{s}\right\}$$
$$= \mathbb{E}[\exp\{\mathrm{i}\langle \dot{X}_\alpha^{\overline{\beta}}(f), \xi\rangle\}].$$

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