

# 南大-上海交大几何分析研讨会

School of Mathematics & IMS, Nanjing University

2024 年 10 月 18 日 – 10 月 21 日

## 一、会议时间与地点：

- 2024 年 10 月 18 日，参会人员直接去酒店办理入住。
- 2024 年 10 月 19 日-10 月 20 日上午，会议学术报告；10 月 20 日下午，自由讨论。
- 2024 年 10 月 21 日，离会。

会议地点：南京大学鼓楼校区（汉口路 22 号）数学学院西大楼 108 室。

## 二、住宿与交通：

住宿：南京市中山路 251 号，新纪元大酒店。

交通方式：

- 飞机抵达。南京禄口机场离新纪元大酒店约 43 公里，1 小时车程，打车费超过 200 元。也可以地铁前往酒店：禄口机场站乘坐地铁 S1 号线至南京南站，换乘地铁 1 号线至珠江路站（1 号口或 3 号口出站），步行 350 米抵达新纪元大酒店，全程约 1.5 小时。
- 高铁抵达。绝大多数高铁会抵达南京南站，建议地铁前往酒店：南京南站乘坐地铁 1 号线至珠江路站（1 号口或 3 号口出站），步行 350 米抵达酒店，全程约半小时，若坐出租车前往也需要半小时，堵车风险较高。
- 火车抵达。少数高铁、动车会抵达南京站，建议地铁前往酒店：南京站乘坐地铁 1 号线至珠江路站（1 号口或 3 号口出站），步行 350 米抵达酒店，全程约 25 分钟，若坐出租车前往需要 20 分钟左右，遇高峰期有堵车风险。

## 三、报告日程安排：

|             | 10 月 19 日 | 10 月 20 日 |
|-------------|-----------|-----------|
| 8:30-9:20   | 李皓昭       | 邱国寰       |
| 9:20-10:10  | 杨晓奎       | 周辉煌       |
| 10:10-10:40 | 茶歇        | 茶歇        |
| 10:40-11:30 | 曾令忠       | 王童瑞       |
| 12:00-14:00 | 午餐        | 午餐        |
| 14:00-14:50 | 王作勤       | 自由讨论      |
| 14:50-15:40 | 王志鑫       |           |
| 15:40-16:10 | 茶歇        |           |
| 16:10-17:00 | 张驰麟       |           |
| 18:00-      | 晚餐        | 晚餐        |

#### 四、报告题目与摘要：(按姓氏拼音排序)

- (1) 李皓昭 (中国科学技术大学)

**Title:** Existence and convergence of Calabi flow

**Abstract:** The Calabi flow, which was introduced by E. Calabi in 1982, is the gradient flow of the Calabi energy and it is a fourth order fully nonlinear parabolic flow of the Kähler potentials. In this talk, I will report some recent results on the long time existence and convergence of Calabi flow.

- (2) 邱国寰 (中国科学院数学与系统科学研究院)

**Title:** A priori interior estimates for special Lagrangian curvature equations

**Abstract:** We establish a priori interior curvature estimates for the special Lagrangian curvature equations in both the critical phase and convex cases. In dimension two, we observe that this curvature equation is equivalent to the equation arising in the optimal transportation problem with a "relative heat cost" function, as discussed in Brenier's paper. When  $0 < \Theta < \frac{\pi}{2}$  (supercritical phase), the equation violates the Ma-Trudinger-Wang condition. So there maybe a singular  $C^{1,\alpha}$  solution in supercritical case which is different from the special Lagrangian equations. We have also demonstrated that these gradient estimates of these curvature equations also hold for all constant phases. It is worth noting that for the special Lagrangian equation, particularly in subcritical phases, the interior gradient estimate remains an open problem. This is joint work with Xingchen Zhou.

- (3) 王童瑞 (上海交通大学)

**Title:** Multiple existence of minimal surfaces with low genus in lens spaces

**Abstract:** In this talk, I will discuss two either-or results for the multiple existences of minimal real projective planes and minimal Klein bottles in certain lens spaces with generic metrics. In particular, we show in positive Ricci  $RP^3$  that there are four distinct minimal real projective planes together with four distinct minimal tori, and the number of minimal tori can be improved to five for almost all metrics of positive Ricci. Our proof is mainly based on a variant multiplicity one theorem for the Simon-Smith min-max theory under certain equivariant settings. This talk is based on the joint work with Xingzhe Li and Xuan Yao.

- (4) 王志鑫 (上海交通大学)

**Title:** Riccati equation for static spaces and its applications

**Abstract:** Static spaces are of paramount importance in both general relativity and differential geometry and have therefore been extensively studied. In this presentation, I will introduce a Riccati-type equation specifically tailored for static spaces and explore its various applications. Riccati-type equations are pivotal in manifold research, particularly for those with a lower Ricci curvature bound, as they lead to significant insights in comparison, splitting, and topological theorems. Intriguingly, a similar Riccati-type equation can be derived for static spaces, offering potential solutions for diverse problems within this domain.

- (5) 王作勤 (中国科学技术大学)

**Title:** On Polya Conjecture for product domains

**Abstract:** In 1954 Polya proposed a conjecture for any Euclidean domain that compares each Dirichlet and Neumann eigenvalue with the leading term in the Weyl asymptotics. In this talk I will review some backgrounds and known results on this conjecture, and discuss our result on this conjecture for thin product type domains/manifolds. This is a joint work with Xiang He.

- (6) 杨晓奎 (清华大学)

**Title:** New characterizations for spheres and projective spaces

**Abstract:** In this talk, we introduce a new positivity notion for curvature of Riemannian manifolds and use it to characterize spherical space forms and the complex projective spaces. Moreover, we present some new volume comparison theorems and rigidity results on compact Kähler manifolds.

- (7) 曾令忠 (江西师范大学)

**Title:** Spectrum of the Dirac operator on compact Riemannian manifolds

**Abstract:** In this talk, we review some progress on the eigenvalue problem of Dirac operator. Furthermore, we would like to introduce some new extrinsic estimates for the sum of arbitrary consecutive  $n$  eigenvalues of Dirac operator acting on some Dirac invariant subbundles and their applications. In addition, we also give some universal bounds under certain curvature condition. Finally, a conjecture shall be proposed. Once this conjecture is proved to be true, a new proof of Willmore conjecture will be provided. This work is jointed with X. Mao.

- (8) 张驰麟 (复旦大学)

**Title:** An area growth estimate of the Liouville equation

**Abstract:** We establish an area growth estimate for solutions that are bounded from above of the Liouville equation  $\Delta u + Ke^{2u} = 0$  with a positive pinched curvature  $0 < \lambda \leq K \leq \Lambda$ . As an application, we provide a new proof of Eremenko-Gui-Li-Xu's result. We also classify solutions with an upper bound in the half plane with the boundary having constant geodesic curvature.

- (9) 周辉煌 (中国科学技术大学)

**Title:** Comparison theorems for fractional GJMS operators

**Abstract:** On the conformal infinity of a Poincare-Einstein manifold, we compare the different fractional order GJMS operators  $P_{2\gamma}$ . We obtain some inequalities between the Yamabe constants and the Green's functions with different  $P_{2\gamma}$ , and prove some rigidity theorems by characterizing the equalities. Furthermore, we prove some positive mass theorems and obtain a Cohn-Vossen type inequality of  $Q_3$  for three-dimensional conformal infinity.