Hausdorff dimension of the Julia sets of some rational maps

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Singular perturbation and McMullen maps

Consider a singular perturbation of the unicritical polynomials

$$f_\lambda(z) = z^q + \frac{\lambda}{z^p}, \text{ where } p \geq 2, q \geq 2, \lambda \in \mathbb{C} \setminus \{0\}.$$

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4. The dynamics of f_λ depends on the **one of** the free critical orbits.

Escape Trichotomy Theorem

Theorem (Devaney, Look and Uminsky, 2005)

Suppose that the free critical points of f_λ are attracted by ∞. Then one and only one of the following three cases happens:

1. $f_\lambda(\omega_j) \in B_\lambda$ *for some* j*, then* J_λ *is a* **Cantor set***;*
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Moreover, if the orbit of ω_j *is bounded, then* $J(f_\lambda)$ *is connected.*

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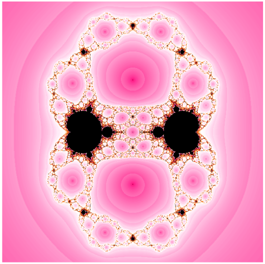
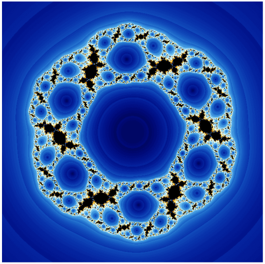
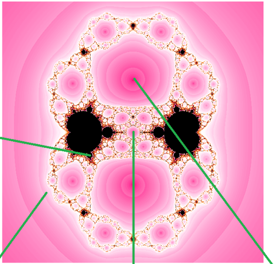


Parameter plane of

$f_\lambda(z) = z^3 + \lambda/z^3$



A connected Julia set



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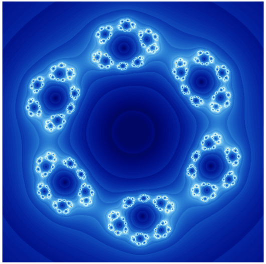

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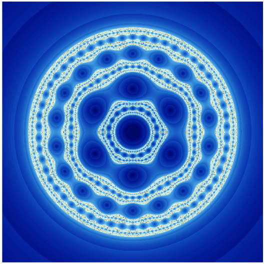

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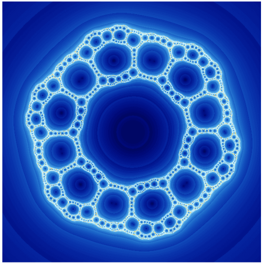

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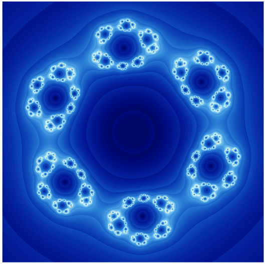
What the values of the Hausdorff dimensions of these three kinds of Julia sets?



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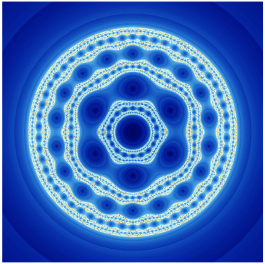

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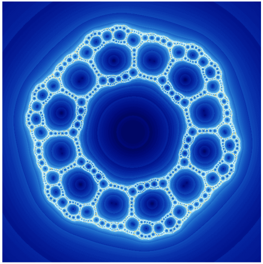

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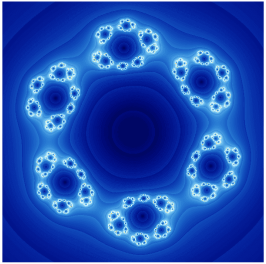

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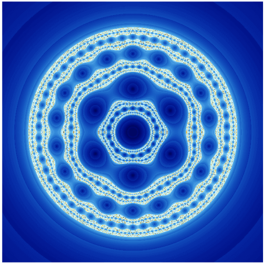

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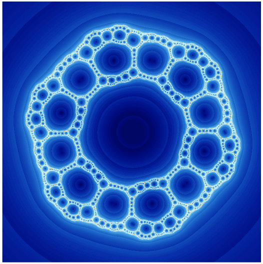

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Main result

Theorem (Y., Qiu-Ren-Y., Fu-Y., 2018)

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Except: We don't know the existence of Cantor Julia sets with positive area.

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There exist cubic polynomials whose Julia sets are Cantor sets having Hausdorff dimension two.

Remark: such Cantor Julia sets have zero area.

Shishikura's criterion

Theorem (Shishikura, 1998)

Suppose that a rational map f_0 of degree $d \geq 2$ has a **parabolic fixed point** *z_0 with multiplier $e^{2\pi i p/q}$ ($p,\ q \in \mathbb{Z}$, $(p,q)=1$) and that the immediate parabolic basin of z_0 contains* **only one critical point** *of f_0. Then for any $\varepsilon > 0$ and $b > 0$, there exist a neighborhood $\mathscr{N}$ of f_0 in the space of rational maps of degree d, a neighborhood V of z_0 in $\widehat{\mathbb{C}}$, positive integers N_1 and N_2 such that if $f \in \mathscr{N}$, and if f has a fixed point in V with multiplier $e^{2\pi i\alpha}$, where*

$$q\alpha = p \pm \cfrac{1}{a_1 \pm \frac{1}{a_2+\beta}}$$

with integers $a_1 \geq N_1$, $a_2 \geq N_2$ and $\beta \in \mathbb{C}$, $0 \leq \operatorname{Re}\beta < 1$, $|\operatorname{Im}\beta| \leq b$, then

$$\dim_H(J(f)) > 2-\varepsilon.$$

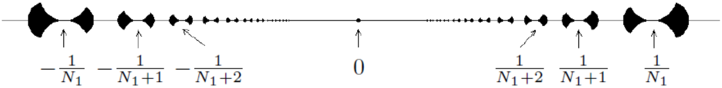

Figure: The domain of α, where $p=0$ and $q=1$.

Branner-Hubbard's characterization on cubic poly.

$$P_{a,b}(z) = z^3 - 3a^2z + b$$

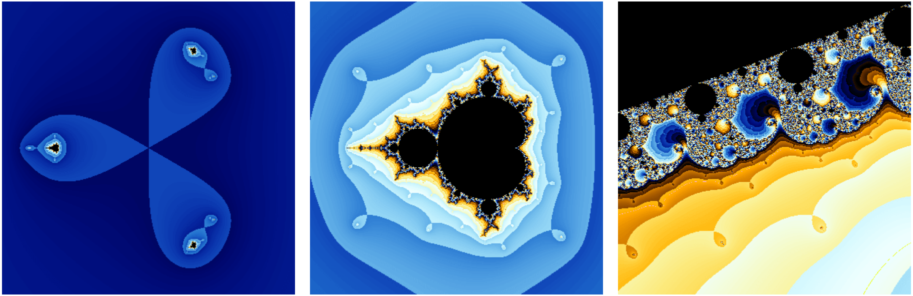

Figure: The space $\mathscr{L}^+(\zeta)$ for some $\zeta > 1$. The set $\mathscr{B}^+(\zeta) \subset \mathscr{L}^+(\zeta)$ has been drawn and zoomed in several times. The **copies of the Mandelbrot set** and some decorations of the **point components** of $\mathscr{B}^+(\zeta)$ can be seen clearly.

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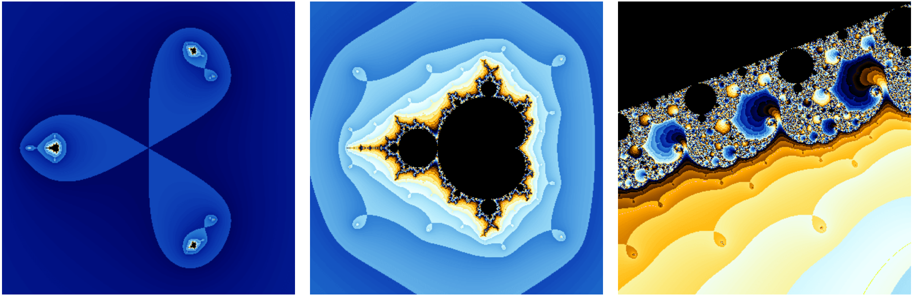

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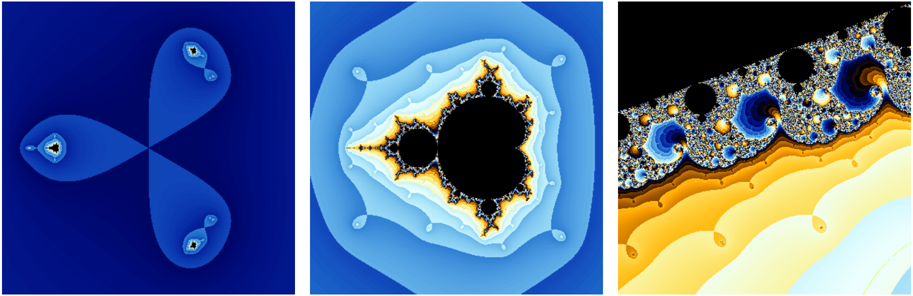

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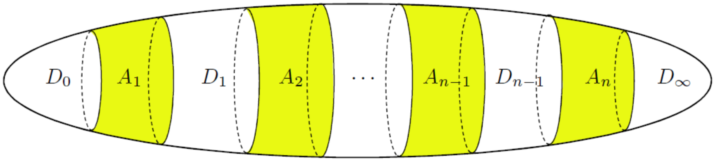
5. Haïssinsky-Pilgrim (2012): The lower bound α of $\dim_H$(Cantor circle Julia sets) is determined by the combinatorics.



$\sum_{i=1}^{n} \left(\frac{1}{d_i}\right)^{\alpha} = 1$, where $\sum_{i=1}^{n} \frac{1}{d_i} < 1$, and α is the conformal dimension

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Theorem (Qiu-Y., 2018)

Let $\mathscr{H}$ be a hyperbolic component containing a rational map f_0 whose Julia set $J(f_0)$ is a Cantor set of circles. Then

$$\inf_{f\in\mathscr{H}} \dim_H(J(f)) = \dim_C(J(f_0)) \quad \textit{and} \quad \sup_{f\in\mathscr{H}} \dim_H(J(f)) = 2.$$

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Theorem (Qiu-Ren-Y., 2018)

Let $f_\lambda(z) = z^q + \lambda/z^p$, where $1/p + 1/q < 1$. Then

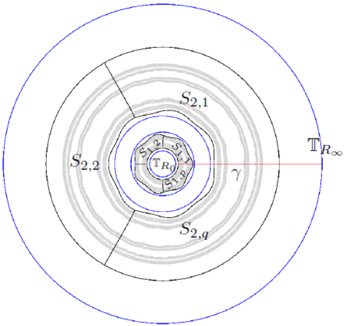
$$\lim_{\lambda\to 0} \dim_H(J(f_\lambda)) = 1 + \alpha_{p,q},$$

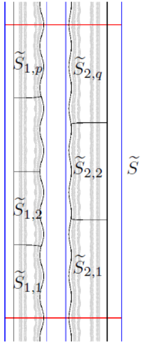
where $\alpha_{p,q} = \alpha \in (0,1)$ is the unique positive root of $p^{-\alpha} + q^{-\alpha} = 1$.

Moreover, if $p = q \geq 3$ then

$$\left|\dim_H(J(f_\lambda)) - \Big(1 + \frac{\log 2}{\log p}\Big)\right| \leq \frac{2^{2p+1}\log(2p)}{\log^2 p}|\lambda|^{1-\frac{2}{p}} + O\big(|\lambda|^{2(1-\frac{2}{p})}\big).$$

The proof idea





Ingredients in the proof:

1. Decompose the dynamics of f_λ to an IFS;
2. Koebe's distortion theorem on the estimation of contraction factors;
3. Falconer's criterion on the Hausdorff dimension of the attractor of the IFS;
4. Put the calculation on the logarithm plane.

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Sierpinski carpet Julia sets

Some known results:

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Theorem (Fu-Y., 2018)

Let $\mathscr{H}$ be a Sierpinski carpet hyperbolic component (actually holds for the hyperbolic Julia sets with a simply connected attracting basin). Then

$$\sup_{f\in\mathscr{H}} \dim_H(J(f)) = 2.$$

Sierpinski carpet Julia sets

Theorem (Y.-Yin, 2018, a refinement of Shishikura's result)

There exist **non-renormalizable** *quadratic polynomials whose periodic points are all repelling and whose Julia sets have Hausdorff dimension two. Moreover, such parameters are dense on the boundary of the Mandelbrot set.*

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Theorem (Fu-Y., 2018)

There exist Sierpinski carpet Julia sets with zero area but with Hausdorff dimension two.

Related topics

One may consider the Lebesgue area and the Hausdorff dimension of some special Julia sets (or subsets):

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Conjecture

For each hyperbolic component $\mathscr{H}$ in the space of rational maps with degree at least two,

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