Hausdorff dimension of irrational indifferent attractors

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Indifferent fixed points

Consider the holomorphic germ

$$f(z) = \lambda z + a_2 z^2 + \cdots, \text{ where } \lambda \in \mathbb{C} \text{ with } |\lambda| = 1.$$

The local dynamics of f near 0 depends on λ:

1. $\lambda = e^{2\pi i\alpha}$ with $\alpha \in \mathbb{Q}$ (rational indifferent): Parabolic point;
2. $\lambda = e^{2\pi i\alpha}$ with $\alpha \in \mathbb{R} \setminus \mathbb{Q}$ (irrational indifferent): Siegel disk or Cremer point.

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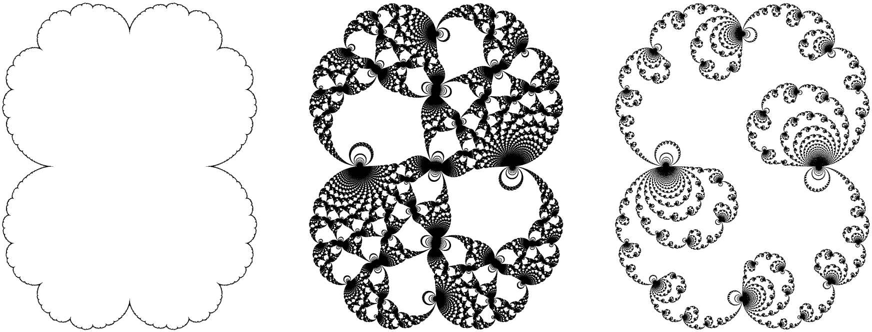
In order to study the irrational indifferent case, an idea is to consider the **perturbations** of rational indifferent.

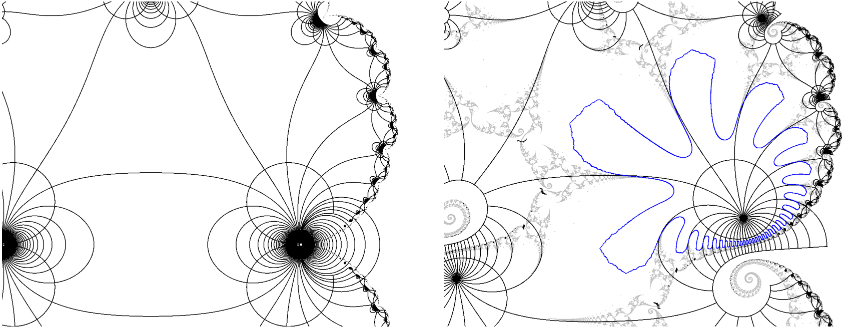
Phenomenon

It was known that the Julia set **does not depend continuously** at the parabolic parameters. One of the interesting phenomenon during the perturbation is **parabolic implosion**.



Phenomenon

Although the Julia set **does not depend continuously** at the parabolic parameters, it turns out that the (disturbed) Fatou coordinate does (restricted on some truncated chessboard).



Developments

The main tools to analyze such bifurcation are **Fatou coordinates** and **horn maps**, which were developed by:

1. Douady-Hubbard (1984-85): landing of external rays at the M-set (Orsay notes), the straightening of polynomial-like maps;
2. Lavaurs (1989): the non-local connectivity of the connectedness locus of cubic polynomials (Ph.d thesis);
3. Douady (1994): the discontinuity of the Julia sets;
4. Shishikura (1998): the Hausdorff dim of ∂M (an invariant class);
5. Yampolsky (2003): cylinder renormalization for critical circle maps;
6. Inou-Shishikura (2006): near-parabolic renormalization (a new invariant class).

Two recent remarkable results

Application of **parabolic implosion**:

Theorem (Astorg-Buff-Dujardin-Peters-Raissy, Ann. Math. 2016)

There exist 2-dimensional polynomial mappings having wandering domains.

Application of (**near-**) **parabolic renormalization**:

Theorem (Buff-Chéritat, Ann. Math. 2012)

There exist quadratic Julia sets with positive area. (Siegel, Cremer, infinitely satellite renormalizable)

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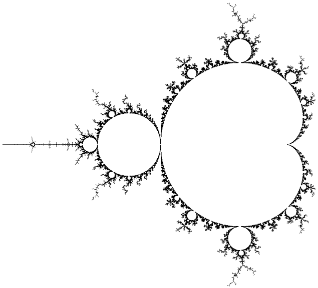
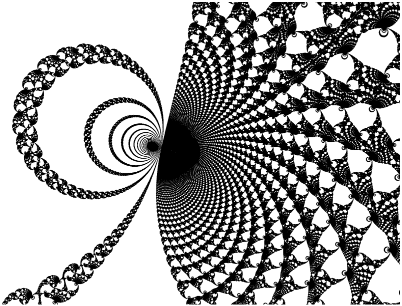
Before stating the basic scheme, let's recall ...

Hausdorff dim of the the boundary of M-set

Denote $P_c(z) = z^2 + c$, where $c \in \mathbb{C}$. The **Mandelbrot set** is defined as

$$\mathrm{M} := \{c \in \mathbb{C} : \lim_{n\to\infty} P_c^{\circ n}(0) \neq \infty\}.$$





Theorem (Shishikura, Ann. Math. 1998)

$\text{H-dim}(\partial \mathrm{M}) = 2.$

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Idea of the proof:

1. perturb parabolic periodic points;
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A class was defined:

$$\mathscr{F}_0 = \left\{ f : Dom(f) \to \mathbb{C} \;\middle|\; \begin{array}{l} 0 \in Dom(f) \text{ open} \subset \mathbb{C}, f \text{ is holo. in } Dom(f), \\ f(0) = 0,\ f'(0) = 1, f : Dom(f) \setminus \{0\} \to \mathbb{C}^* \text{ is a} \\ \text{branched covering with a unique critical value} \\ cv_f, \text{ all critical points are of local degree 2} \end{array} \right\}.$$

The class satisfies $\mathscr{R}_0(\mathscr{F}_0) \subset \mathscr{F}_0$, where $\mathscr{R}_0$ is **parabolic renormalization** operator.

Inou-Shishikura's class

Let $P(z) = z(1+z)^2$. Then P has a parabolic fixed point at 0 and **two simple** critical points -1 and $\mathrm{cp}_P = -\frac{1}{3}$ with $P(-1) = 0$ and $\mathrm{cv}_P = P(\mathrm{cp}_P) = -\frac{4}{27}$. Let V be a Jordan domain of $\mathbb{C}$ containing 0 and define

$$IS_0 := \left\{ f = P \circ \varphi^{-1} : \varphi(V) \to \mathbb{C} \;\middle|\; \begin{array}{l} \varphi : V \to \mathbb{C} \text{ is univalent,} \\ \varphi(0) = 0,\ \varphi'(0) = 1 \text{ and} \\ \varphi \text{ has a q.c. extension to } \mathbb{C} \end{array} \right\}.$$

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For $\alpha\in\mathbb{R}$, define

$$IS_\alpha=\{f(z)=f_0(e^{2\pi i\alpha}z): e^{-2\pi i\alpha}\cdot Dom(f_0)\to\mathbb{C}\mid f_0\in IS_0\}.$$

For all $f\in IS_\alpha$, the critical value is always $\mathrm{cv}=-\frac{4}{27}$.

The renormalization operator is hyperbolic

Let N be a positive integer. Denote

$$\mathrm{HT}_N := \{\alpha = [0; a_1, a_2, \cdots] \in (0,1) \setminus \mathbb{Q} \mid a_n \geq N \text{ for all } n \geq 1\}.$$

Theorem (Inou-Shishikura, 2006)

There are two Jordan domains V and V' satisfying $V \Subset V'$ and a number ε_0 such that for all $f \in IS_\alpha$ with $\alpha \in (0, \varepsilon_0]$, then $\mathscr{R}f$ is well-defined so that

1. *$\mathscr{R}f = P \circ \psi^{-1} \in IS_{1/\alpha}$. Moreover, ψ extends to a univalent function from V' to $\mathbb{C}$. In particular, if $\alpha \in \mathrm{HT}_N$ for $N \geq 1/\varepsilon_0$, then $\mathscr{R}$ can be iterated infinitely many times.*

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2. *There exists a constant $0 < \nu < 1$, such that for all $f, g \in IS_\alpha$ with $\alpha \in (0, \varepsilon_0]$, the operator $\mathscr{R}$ is a* **uniform contraction in the fiber direction***:*

$$\mathrm{d}_{\mathrm{Teich}}(\pi \circ \mathscr{R}(f), \pi \circ \mathscr{R}(g)) \leq \nu \, \mathrm{d}_{\mathrm{Teich}}(\pi(f), \pi(g)),$$

where $\pi : IS_\beta \to IS_0$ is the projection defined by $\pi(h) = h(e^{-2\pi i \beta} z)$.

Some remarkable applications

for $z^2 + c$ and high type

Area of the Julia sets and post-critical sets:

1. Buff-Chéritat (Ann. Math. 2012): Quadratic Julia sets with positive area.
2. Avila-Lyubich (arXiv 2015): Quadratic Feigenbaum Julia sets with positive area.
3. Cheraghi (CMP 2013): Zero area of post-critical set for Brjuno.
4. Cheraghi (Ann. Sci. École Norm. Sup. 2019): zero area of post-critical set for non-Brjuno.

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Topology of the post-critical sets:

1. Shishikura-Y. (arXiv 2016): Douady-Sullivan's conjecture and Herman's conjecture.
2. Cheraghi (arXiv 2017): Topology of the post-critical sets (hedgehogs).
3. Cheraghi-Pedramfar (preprint 2018): Complex Feigenbaum phenomena.

Some remarkable applications

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Other applications:

1. Cheraghi-Chéritat (Invent. Math. 2015): Marmi-Moussa-Yoccoz conjecture.
2. Cheraghi-Shishikura (arXiv 2015): MLC at unbounded type infinitely satellite renormalization pts.
3. Avila-Cheraghi (JEMS 2018): Statistical properties (uniquely ergodic on the post-critical set) and small cycles.
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Remark: Recently, Dudko, Lyubich and Selinger (arXiv 2017, 2018) developed a theory called "Pacman renormalization" (combines features of two classical Renormalization Theories: Quadratic-like and Siegel), which can be used to prove

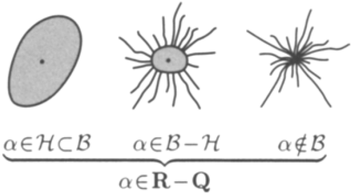
1. The self-similarity of the Mandelbrot set near Siegel parameters.
2. MLC at some bounded type infinitely satellite renormalization pts.
3. The local connectivity of the infinitely satellite renormalization Julia sets.
4. ∃ some bounded type infinitely satellite renormalization Julia sets with positive area.

Hedgehogs

Let f be a non-linear holomorphic system with the form

$$f(z) = e^{2\pi i\alpha} z + \mathcal{O}(z^2), \text{ where } \alpha \in \mathbb{R} \setminus \mathbb{Q}.$$

Pérez-Marco proved that if f and f^{-1} are defined and univalent in a **neighborhood** of the closure of a Jordan domain $U \subset \mathbb{C}$ containing 0, then there exists a compact, full and connected set $K = K_{f,U}$ contained in $\overline{U}$ such that $0 \in K$, $K \cap \partial U \neq \emptyset$ and $f(K) = f^{-1}(K) = K$.



Siegel compacta: K's
hedgehog: if K is not contained in the closure of a linearization domain.

Topology and geometry of hedgehogs

If ∂U is C^1-smooth, Pérez-Marco proved that

- K is in unique.
- the non-linearizable hedgehogs (i.e. 0 is a Cremer point) have no interior and they are not locally connected at any point different from the fixed point.

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For $P_\alpha(z) = e^{2\pi i\alpha} z + z^2$, it is known that

- (McMullen, 1998) $\dim(\partial\Delta_\alpha) \le \dim_H(J_\alpha) < 2$ if α is of bounded type.
- (Graczyk-Jones, 2002): Δ_α quasicircle and $\text{cp} \in \partial\Delta_\alpha$, then $\dim_H(\partial\Delta_\alpha) > 1$.
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Biswas constructed some non-linearizable hedgehogs of holomorphic germs s.t.

- (2008) they have Hausdorff dimension one.
- (2016) they have positive area.

Main result

For $\alpha \in \mathrm{HT}_N$,

- If $\alpha \in \mathscr{H}$, then $\mathrm{cp} \in \Delta_\alpha$;
- If $\alpha \in \mathscr{B} \setminus \mathscr{H}$, then $\mathrm{cp} \notin \Delta_\alpha$;
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Theorem (Cheraghi-DeZotti-Y., 2018)

There exists $N > 0$ such that for all $\alpha \in \mathrm{HT}_N \setminus \mathscr{H}$ and all $f \in IS_\alpha$, the post-critical set of f has Hausdorff dimension two.

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Corollary

For all $\alpha \in \mathrm{HT}_N \setminus \mathscr{H}$, $J(P_\alpha)$ has Hausdorff dimension two.

Question: Is there any $\alpha \notin \mathscr{H}$ s.t. $Area(J(P_\alpha)) = 0$?

McMullen's criterion

Proposition (McMullen, 1987)

Let $\{\mathscr{K}_n\}_{n=0}^{\infty}$ be a sequence satisfying the **nesting conditions**. Let $\delta_{n+1} > 0$ such that for all $1 \leq i \leq l_n$ and $K_{n,i} \in \mathscr{K}_n$, we have

$$\text{density}(\mathscr{K}_{n+1}, K_{n,i}) := \text{density}\Big(\cup_{j=1}^{l_{n+1}} K_{n+1,j}, K_{n,i}\Big) \geq \delta_{n+1}.$$

Suppose that for each $K_{n,i} \in \mathscr{K}_n$ with $n \geq 1$,

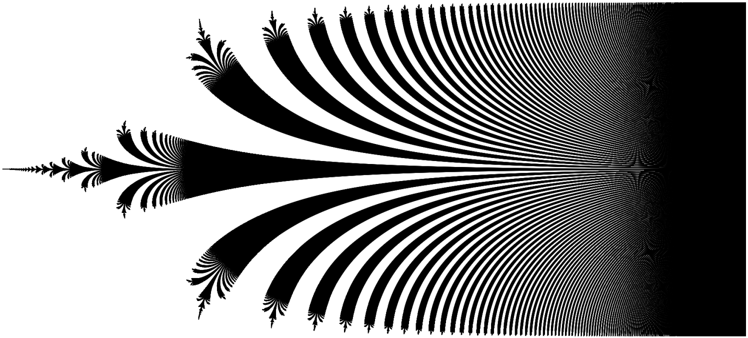
$$\operatorname{diam} K_{n,i} \leq d_n < 1, \text{ where } d_n \to 0 \text{ as } n \to \infty.$$

Then the Hausdorff dimension of $\bigcap_n \mathscr{K}_n$ satisfies

$$\dim_H \Big(\bigcap_{n\in\mathbb{N}} \mathscr{K}_n\Big) \geq 2 - \limsup_{n\to\infty} \frac{\sum_{k=1}^{n+1} |\log \delta_k|}{|\log d_n|}.$$

Result on exponential maps

Application (McMullen, 1987): $\dim_H(J(\lambda e^z)) = 2$, where $0 < \lambda < 1/e$.



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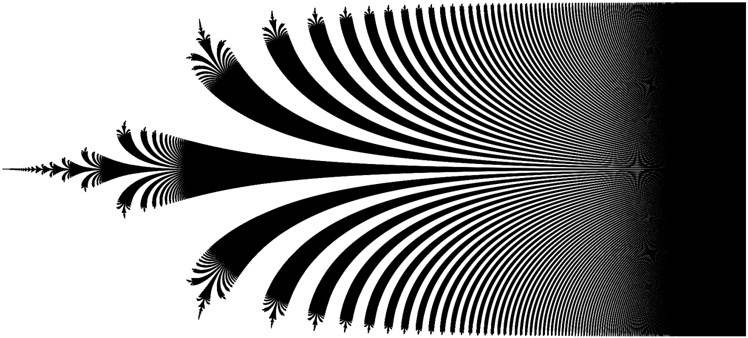
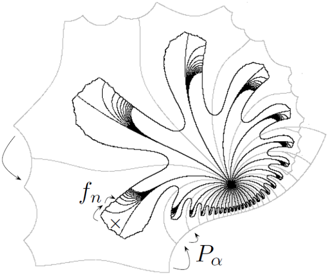


Silva (1988): hairs are C^∞.
Karpińska (1999): $\dim_H(\textit{end points}) = 2$ and $\dim_H(\textit{hairs without end points}) = 1$.

Topology of post-critical sets (maximal hedgehogs)



Cheraghi (2017): For high type rotation numbers,

- If $\alpha \notin \mathscr{B}$, $P(f)$ is a Cantor bouquet;
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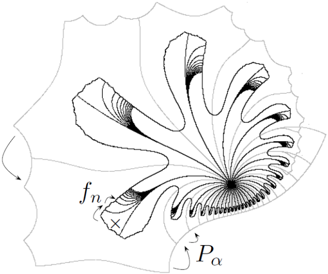

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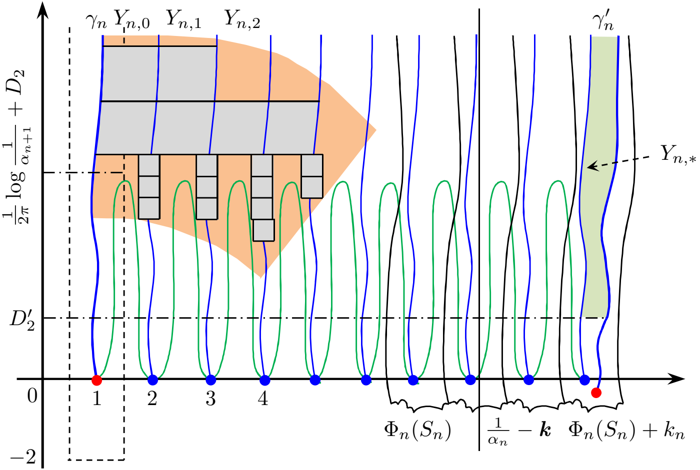
Cheraghi-DeZotti-Y. (2018): For high type rotation numbers,

- There exist $\alpha \notin \mathscr{B}$ and $\alpha \in \mathscr{B} \setminus \mathscr{H}$, such that Karpinska's paradox holds.

Find sets satisfying the nesting conditions



Further developments and questions

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THANK YOU FOR YOUR ATTENTION !