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===========================================================================
*****  Please provide proper reference to
*****     S.M. Rump: INTLAB - INTerval LABoratory. In Tibor Csendes,
*****       editor, Developments in Reliable Computing, pages 77-104.
*****       Kluwer Academic Publishers, Dordrecht, 1999,
*****       http://www.ti3.tuhh.de/intlab .
*****
*****  Commercial use or use in conjunction with a commercial program
*****  which requires INTLAB or part of INTLAB to function properly
*****  requires a special license. Please contact me  rump (at) tuhh.de


>> intvalinit('DisplayInfsup')
===> Default display of intervals by infimum/supremum (e.g. [ 3.14 , 3.15 ])
>>
>>
>> c6=infsup(6,6);
mu=11-4*sqrt(c6)
cp_P=1-2*sqrt(c6)/3
cv_P=-16/(3*(8*sqrt(c6)+3))
v1P=9*sqrt(c6)-47/2-sqrt(996*sqrt(c6)-2439)/2
v2P=9*sqrt(c6)-47/2+sqrt(996*sqrt(c6)-2439)/2
intval mu =
[   1.20204102886728,   1.20204102886729]
intval cp_P =
[  -0.63299316185546,  -0.63299316185545]
intval cv_P =
[  -0.23603083295667,  -0.23603083295666]
intval v1P =
[  -1.87046002590218,  -1.87046002590189]
intval v2P =
[  -1.03872460400091,  -1.03872460400062]
>>
>>
>> cp=(1+4*sqrt(c6)+2*sqrt(2*(9+sqrt(c6))))/5
cpp=(1+4*sqrt(c6)-2*sqrt(2*(9+sqrt(c6))))/5
omega=(8*sqrt(c6)-3)/25+i*(6*sqrt(c6)+4)/25
cv=3*(8*sqrt(c6)+3)/4
v1=(2+v1P)/(-v1P)+i*2*sqrt(-1-v1P)/(-v1P)
v2=(2+v2P)/(-v2P)+i*2*sqrt(-1-v2P)/(-v2P)
intval cp =
[   4.07370692077733,   4.07370692077736]
intval cpp =
[   0.24547666767573,   0.24547666767574]
intval omega =
[   0.66383671769061 +  0.74787753826796i,   0.66383671769062 +  0.747877753826797i]
intval cv =
[  16.94693845669905,  16.94693845669908]
intval v1 =
[   0.06925567630618 +  0.997598943112 34i,   0.06925567630666 +  0.997598943112 82i]
intval v2 =
[   0.92543816936245 +  0.37889866017772i,   0.92543816936445 +  0.37889866017972i]
>>
>>
>> a0=infsup(-0.06,-0.06);
r0=infsup(1.07,1.07);
epsilon1=infsup(0.128,0.128);
```

```
epsilon2=infsup(0.007,0.007);
epsilon3=infsup(0.014,0.014);
>>
>>
>> eta=infsup(3,3);
epsilon1^6*(2-epsilon1)^4/((1+epsilon1)*(2+epsilon1)^8)
cv*exp(-2*pi*eta)
(2-epsilon2)^6*epsilon2^4/((1+epsilon2)*(1+epsilon2)^8)
(2+epsilon3)^6*(1+epsilon3)^4/((1-epsilon3)*epsilon3^4*(1-epsilon3)^4)
cv*exp(2*pi*eta)
(real(omega)+0.06)^2+imag(omega)^2-(r0-epsilon3)^2
intval ans =
  1.0e-006 *
[   0.11386784640381,   0.11386784640382]
intval ans =
  1.0e-006 *
[   0.11036544767480,   0.11036544767481]
intval ans =
  1.0e-006 *
[   0.14130929096480,   0.14130929096481]
intval ans =
  1.0e+009 *
[   1.97066090029752,   1.97066090029754]
intval ans =
  1.0e+009 *
[   2.60225214599211,   2.60225214599213]
intval ans =
[  -0.03187559387713,  -0.03187559387712]
>>
>>
>> a4=infsup(-0.22+0.69*i,-0.22+0.69*i);
epsilon4=abs(a4-omega);
a5= infsup(0.78+0.21*i,0.78+0.21*i);
epsilon5=abs(a5-omega);
a6=omega;
epsilon6=infsup(0.41,0.41);
epsilon7=infsup(0.82,0.82);
r1=infsup(1.2,1.2);
>>
>>
>> addpath('C:\Users\yangfei\Desktop\functions'); %add function file
>>
>>
>> x60=infsup(0.54,0.54);
x61=real(omega);
x62p=infsup(1.07,1.07);
x62m=infsup(1.067,1.067);
x63=real(omega)+epsilon6;

(x60-real(omega))^2+(y4p(x60)-imag(omega))^2-epsilon6^2
(x60-real(a4))^2+(y6p(x60)-imag(a4))^2-epsilon4^2
(x62p-real(omega))^2+(y5p(x62p)-imag(omega))^2-epsilon6^2
(x62m-real(a5))^2+(y6m(x62m)-imag(a5))^2-epsilon5^2
intval ans =
[   0.00484765552797,   0.00484765552798]
intval ans =
[  -0.00555989422406,  -0.00555989422405]
intval ans =
[   0.00179907272648,   0.00179907272649]
```

```
intval ans =
[  -0.00577509552096,  -0.00577509552094]
>>
>>
>> y4p(x60)-23*(x60+1)/26
x601=infsup(0.6,0.6);
Xi4p(x60,x601)
Xi4p(x601,x61)
intval ans =
[  -0.21742622096150,  -0.21742622096149]
intval ans =
  1.0e+002 *
[   1.40640535177714,   1.40640535177727]
intval ans =
  1.0e+002 *
[   2.17073752473541,   2.17073752473624]
>>
>>
>> s61=imag(a5+omega)/real(a5-omega);
real(a5)+epsilon5/sqrt(1+s61^2)
imag(a5)-imag(omega)*(real(a5)-1)/(real(omega)-1)
x611=infsup(0.99,0.99);

xi55pmax=(sqrt((real(a5-omega))^2+(imag(a5+omega))^2)+epsilon5)^4;
xi51p(x61)*xi52p(x611)/( xi53p(x611)*xi54p(x611)*xi55pmax)

x612=infsup(1.05,1.05);
x613=infsup(1.06,1.06);

HatXi5p(x611,x612)
HatXi5p(x612,x613)
HatXi5p(x613,x62p)
intval ans =
[   0.84624772243093,   0.84624772243094]
intval ans =
[  -0.27944387170616,  -0.27944387170614]
intval ans =
  1.0e+002 *
[   1.32594990847299,   1.32594990847306]
intval ans =
  1.0e+002 *
[   1.37608951410455,   1.37608951410462]
intval ans =
  1.0e+002 *
[   1.41476217224043,   1.41476217224050]
intval ans =
  1.0e+002 *
[   1.26547650628301,   1.26547650628308]
>>
>>
>> s62=imag(omega)/(real(omega)+1);
real(omega)+epsilon6/sqrt(1+s62^2)
s63=imag(omega)/(real(omega)-1);
real(omega)-epsilon6/sqrt(1+s63^2)
s64=imag(omega)/real(omega);
real(omega)+epsilon6/sqrt(1+s64^2)

x621=real(omega);
x622=0.99;
```

```
x623=1.07;
x63=real(omega)+epsilon6;

xi61p(x60)*xi62p(x621)/(xi63p(x621)*epsilon6^4*xi65p(x621))
xi61p(x621)*xi62p(x622)/((1+epsilon6)*epsilon6^4*xi65p(x621))
xi61p(x622)*xi62p(x623)/(xi63p(x622)*epsilon6^4*xi65p(x622))
xi61p(x623)*xi62p(x623)/(xi63p(x622)*epsilon6^4*xi65p(x622))

((x63+1)^2+(imag(omega))^2)^3*((x63-1)^2+(imag(omega))^2)^2/(xi63p(x623)↙
*epsilon6^4*xi65p(x623))

x631=1.069;
x632=1.072;

Xi6m(x631,x62m)
Xi6m(x632,x631)
xi61m(x632)*xi62m(x632)/((x63^2+(imag(omega))^2)^(1/2)*epsilon6^4*((x63-real(omega))↙
^2+(2*imag(omega))^2)^2)
intval ans =
[   1.03779590814045,   1.03779590814046]
intval ans =
[   0.49574589736391,   0.49574589736392]
intval ans =
[   0.93600977194376,   0.93600977194378]
intval ans =
  1.0e+002 *
[   2.09680088106572,   2.09680088106577]
intval ans =
  1.0e+002 *
[   1.30094023720587,   1.30094023720592]
intval ans =
  1.0e+002 *
[   1.30879579829141,   1.30879579829151]
intval ans =
  1.0e+002 *
[   1.29168673192868,   1.29168673192878]
intval ans =
  1.0e+002 *
[   1.46282281027926,   1.46282281027934]
intval ans =
  1.0e+002 *
[   1.25399801206324,   1.25399801206336]
intval ans =
  1.0e+002 *
[   1.26550600983070,   1.26550600983085]
intval ans =
  1.0e+002 *
[   1.33076348007783,   1.33076348007797]
>>
>>
>> x70=-1-epsilon7;
x71=-1.095;
x72=real(a4)-epsilon4
x73=-0.77;

(x71-real(a4))^2+(y7p(x71)-imag(a4))^2-epsilon4^2
(x71+1)^2+(y4p(x71))^2-epsilon7^2
y4m(x73)
```

```
s71=-imag(omega)/(real(omega)+1);
-1-epsilon7/sqrt(1+s71^2)

x701=-1.3;
(x70+1)^6*(x70-1)^4/(xi73p(x701)*xi74p(x701)*((x70-real(omega))^2+(imag(omega))^2)^2)
(x70+1)^6*(x70-1)^4/(xi73p(x701)*xi74p(x701)*xi75p(x701))
Xi7p(x71,x701)

s72=imag(a4)/(real(a4)-1);
real(a4)-epsilon4/sqrt(1+s72^2)

xi41p(x71)*xi42p(x71)/((x72^2+(imag(a4))^2)^(1/2)*xi44p(x71)*((x72-real(omega))^2+↙
(imag(a4)+imag(omega))^2)^2)

s73=imag(a4)/(real(a4)+1);
real(a4)-epsilon4/sqrt(1+s73^2)

s74=imag(omega-a4)/real(omega-a4);
real(a4)-epsilon4/sqrt(1+s74^2)

x721=-1;

((x72+1)^2+(imag(a4))^2)^3*((x72-1)^2+(imag(a4))^2)^2/(xi43m(x721)* ((x72-real↙
(omega))^2+(imag(a4)-imag(omega))^2)^2*xi45m(x721))

((x72+1)^2+(imag(a4))^2)^3*((x72-1)^2+(imag(a4))^2)^2/(xi43m(x721)*xi44m(x721)*xi45m↙
(x721))
Xi4m(x73,x721)
xi41m(x73)*xi42m(x721)/(xi43m(x73)*xi44m(x73)*xi45m(x73))
intval x72 =
[  -1.10572972907885,  -1.10572972907884]
intval ans =
[  -0.00339729069895,  -0.00339729069894]
intval ans =
[   0.02129639834331,   0.02129639834334]
intval ans =
[  -0.00427455158179,  -0.00427455158177]
intval ans =
[  -1.74791838089968,  -1.74791838089967]
intval ans =
[   0.01893617718805,   0.01893617718806]
intval ans =
[   0.02274289545324,   0.02274289545325]
intval ans =
[   0.02613246307297,   0.02613246307298]
intval ans =
[  -0.99096566278103,  -0.99096566278102]
intval ans =
[   0.02538634716629,   0.02538634716630]
intval ans =
[  -0.88340852237560,  -0.88340852237558]
intval ans =
[  -1.10383671769062,  -1.10383671769061]
intval ans =
[   0.01892703739895,   0.01892703739896]
intval ans =
[   0.02071476008895,   0.02071476008896]
intval ans =
  1.0e-003 *
```

```
[   0.17978282899864,   0.17978282899867]
intval ans =
  1.0e-004 *
[   0.68254862933448,   0.68254862933450]
>>
>>
>> x4=a0+r0*r1;
x1=-1.1;
x2=0.54;
x3=1.03;

(x1+1)^2+(yy(x1))^2-epsilon7^2

(x1-real(a4))^2+(yy(x1)-imag(a4))^2-epsilon4^2
(x2-real(a4))^2+(yy(x2)-imag(a4))^2-epsilon4^2

(x2-real(omega))^2+(yy(x2)-imag(omega))^2-epsilon6^2
(x3-real(omega))^2+(yy(x3)-imag(omega))^2-epsilon6^2

(x3-real(a5))^2+(yy(x3)-imag(a5))^2-epsilon5^2
(x4-real(a5))^2+(imag(a5))^2-epsilon5^2

x1p=-0.5;
x2p=0.3;

(x1p-real(a4))^2+(imag(a4))^2-epsilon4^2
(x2p-real(a4))^2+(imag(a4))^2-epsilon4^2
(x2p-real(a5))^2+(imag(a5))^2-epsilon5^2
intval ans =
[  -0.09534400000001,  -0.09534399999999]
intval ans =
[  -0.00614421981073,  -0.00614421981071]
intval ans =
[  -0.00872314868316,  -0.00872314868315]
intval ans =
[  -0.00275363901174,  -0.00275363901173]
intval ans =
[  -0.02923099660525,  -0.02923099660524]
intval ans =
[  -0.02068011486057,  -0.02068011486055]
intval ans =
[  -0.06157015433010,  -0.06157015433009]
intval ans =
[  -0.23001715297409,  -0.23001715297408]
intval ans =
[  -0.03801715297409,  -0.03801715297408]
intval ans =
[  -0.02830615433010,  -0.02830615433009]
>>
>>
>> b0=2*(13+32*sqrt(c6))/25
b1=(2029+256*sqrt(c6))/125

a11=2*(617+688*sqrt(c6));
a01=25*(119+16*sqrt(c6));
a12=3889250+837000*sqrt(c6);
a02=2755539+487396*sqrt(c6);
a13=31356325+8965425*sqrt(c6);
a03=66811702+23697378*sqrt(c6);
```

```
a14=102142212+38104768*sqrt(c6);
a04=240990025+94826600*sqrt(c6);
intval b0 =
[   7.31069374152493,   7.31069374152494]
intval b1 =
[  21.24855499321994,  21.24855499321996]
>>
>>
>> Q2max(11)
64*(617+688*sqrt(c6))/3125
LogDQmax(5.6)
intval ans =
[   0.29980316960524,   0.29980316960525]
intval ans =
[  47.15005835335323,  47.15005835335326]
intval ans =
[   1.47600157157429,   1.47600157157431]
>>
>>
>> c00=infsup(0.06,0.06);
c01max=infsup(2.14,2.14);
>>
>>
>> c11=infsup(11,11);
log(c11)+0.06*pi/3
14*sqrt(c6)/25+0.45
intval ans =
[   2.46072712587016,   2.46072712587017]
intval ans =
[   1.82171425595857,   1.82171425595859]
>>
>>
>> alpha1=atan(imag(a5)/(real(a5)-a0))
alpha2m=infsup(0.54,0.54);
alpha2p=infsup(0.55,0.55);
alpha3=atan(imag(omega)/(real(omega)-a0))
alpha4=atan((imag(omega)+epsilon6)/(real(omega)-a0))

h0=14*sqrt(c6)/25+0.45;
alpha5=pi-atan((h0+1)/(a0+1))
intval alpha1 =
[   0.24497866312686,   0.24497866312687]
intval alpha3 =
[   0.80173196085049,   0.80173196085050]
intval alpha4 =
[   1.01209562738760,   1.01209562738761]
intval alpha5 =
[   1.89236461325771,   1.89236461325772]
>>
>>
>> r5(0)-r2(alpha1)
(a0+r0*r5(alpha2p)*cos(alpha2p)-real(omega))^2 +(r0*r5(alpha2p)*sin(alpha2p)-imag↙
(omega))^2-epsilon6^2
r5(alpha2p)-r2(alpha2p)

r6(alpha4)-r2(alpha4)
(a0+r0*r6(alpha2m)*cos(alpha2m)-real(a5))^2 +(r0*r6(alpha2m)*sin(alpha2m)-imag(a5))↙
^2-epsilon5^2
r6(alpha2m)-r2(alpha3)
```

```
intval ans =
[   0.14353122505575,   0.14353122505577]
intval ans =
[  -0.00917071450962,  -0.00917071450961]
intval ans =
[   0.06316437307283,   0.06316437307285]
intval ans =
[   0.00629601051149,   0.00629601051153]
intval ans =
[  -0.00317601910877,  -0.00317601910874]
intval ans =
[   0.01527910994385,   0.01527910994388]
>>
>>
>> tt4=ones(1,24);
for k = 2:10
  tt4(k)=1+0.01*k;
end

for k = 11:15
  tt4(k)=1.1+0.02*(k-10);
end

for k = 16:23
  tt4(k)=1.2+0.05*(k-15);
end

r7(alpha4)-r2(tt4(2))
for k = 2:22
  r7(tt4(k))-r2(tt4(k+1))
end
r7(tt4(23))-r2(alpha5)
intval ans =
[   0.00386269642522,   0.00386269642523]
intval ans =
[   0.00311999972726,   0.00311999972728]
intval ans =
[   0.00309358830311,   0.00309358830313]
intval ans =
[   0.00317073994419,   0.00317073994420]
intval ans =
[   0.00335259261728,   0.00335259261729]
intval ans =
[   0.00364036227344,   0.00364036227345]
intval ans =
[   0.00403534509713,   0.00403534509714]
intval ans =
[   0.00453891989777,   0.00453891989778]
intval ans =
[   0.00515255065049,   0.00515255065051]
intval ans =
[   0.00238123779397,   0.00238123779398]
intval ans =
[   0.00408058968341,   0.00408058968342]
intval ans =
[   0.00624433982898,   0.00624433982899]
intval ans =
[   0.00888825940827,   0.00888825940828]
intval ans =
```

```
[   0.01202984025661,   0.01202984025662]
intval ans =
[   0.00338905066505,   0.00338905066506]
intval ans =
[   0.01407774098439,   0.01407774098440]
intval ans =
[   0.02845769936545,   0.02845769936546]
intval ans =
[   0.04702287002515,   0.04702287002516]
intval ans =
[   0.07039891324057,   0.07039891324059]
intval ans =
[   0.09937977760593,   0.09937977760594]
intval ans =
[   0.13497774615090,   0.13497774615091]
intval ans =
[   0.17849286242455,   0.17849286242456]
intval ans =
[   0.00833083292375,   0.00833083292377]
>>
>>
>> theta51=infsup(2.38,2.38);
theta52=infsup(2.6,2.6);
r8(alpha5)-r2(theta51)
r8(theta51)-r2(theta52)
intval ans =
[   0.02779813700724,   0.02779813700726]
intval ans =
[   0.03885264547485,   0.03885264547487]
>>
>>
>> c00+c01max+varphi1max(11)
(cv-11)*sin(pi/6)
intval ans =
[   2.30490423435328,   2.30490423435329]
intval ans =
[   2.97346922834953,   2.97346922834954]
>>
>>
>> b0-c00-b1/(2*6.1)
betamax(6.1)
intval ans =
[   5.50900890601510,   5.50900890601511]
intval ans =
[   5.50467057792381,   5.50467057792383]
>>
>>
>> c00+c01max+phi1max(cv-2.35)
2.35/exp(-LogDvarphimax(cv-2.35-3.5))
2.4*exp(LogDQmax(cv-2.4))

2.75+Q(cv)-25.5
infsup(1,1)*(25.5-22)*sin(7*pi/20)
b1/20+Q2max(20)
intval ans =
[   3.48325491783422,   3.48325491783423]
intval ans =
[   2.37229663406748,   2.37229663406749]
intval ans =
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```
[   2.70849870606246,   2.70849870606247]
intval ans =
[   2.83599984045100,   2.83599984045136]
intval ans =
[   3.11852283465928,   3.11852283465929]
intval ans =
[   1.13671786028116,   1.13671786028117]
>>
>>
>> (phi1max(125)+b1/122+Q2max(122))/5
LogDFmax(5*25)
intval ans =
[   0.06448049609418,   0.06448049609419]
intval ans =
[   0.00145943721585,   0.00145943721586]
>>
>>
>> theta1=infsup(3*pi/20,3*pi/20);
theta2=infsup(pi/4,pi/4);
u1theta1=infsup(8.5,8.5);
u2theta1=infsup(6.1,6.1);
u3=infsup(22,22)*cos(theta1);
u4=infsup(17.3,17.3);
u1theta2=infsup(9,9);
u2theta2=infsup(6.6,6.6);

u0theta1=u1theta1/cos(theta1)
u0theta2=u1theta2/cos(theta2)

u2theta1+c00*cos(theta1)+c01max+varphi1max(u2theta1)
u2theta2+c00*cos(theta2)+c01max+varphi1max(u2theta2)

u4+c00*cos(theta1)+c01max+varphi1max(u4)
u3

ArgDeltaFmax(u2theta1,theta1)
ArgDeltaFmax(u2theta1,-theta1)
-ArgDeltaFmin(u2theta1,theta1)
-ArgDeltaFmin(u2theta1,-theta1)

ArgDeltaFmax(u2theta2,theta2)
ArgDeltaFmax(u2theta2,-theta2)
-ArgDeltaFmin(u2theta2,theta2)
-ArgDeltaFmin(u2theta2,-theta2)

pi/2-theta2
intval u0theta1 =
[   9.53977301989206,   9.53977301989207]
intval u0theta2 =
[  12.72792206135785,  12.72792206135786]
intval ans =
[   8.48452638180902,   8.48452638180903]
intval ans =
[   8.95867636590075,   8.95867636590077]
intval ans =
[  19.55993399135539,  19.55993399135541]
intval u3 =
[  19.60214353214409,  19.60214353214410]
intval ans =
```

```
[   0.58278335743794,   0.58278335743795]
intval ans =
[   0.76195694106910,   0.76195694106911]
intval ans =
[   0.76195694106910,   0.76195694106911]
intval ans =
[   0.58278335743794,   0.58278335743795]
intval ans =
[   0.50699611304471,   0.50699611304472]
intval ans =
[   0.77671952010329,   0.77671952010331]
intval ans =
[   0.77671952010329,   0.77671952010331]
intval ans =
[   0.50699611304471,   0.50699611304472]
intval ans =
[   0.78539816339744,   0.78539816339745]
>>
>>
>> r4=infsup(0.34,0.34);
u5=u3-u1theta1;

abs(b0-c00+b1*exp(-theta1*i)/(2*u4)-2*u5*r4^2*exp(theta1*i)/(1-r4^2))+betamax(u4)-2 ↙
*u5*r4/(1-r4^2)

-ArgDeltaFmin(u4,theta1)+LogDFmax(u4)/2-log(1-r4^2)/2
pi/infsup(5,5)

-ArgDeltaFmax(u4,theta1)-LogDFmax(u4)/2+log(1-r4^2)/2
-3*pi/infsup(20,20)

exp(LogDFmax(u4)/2)/(AbsDeltaFmin(u4,theta1)*sqrt(1-r4^2))
sqrt(1-r4^2)/(AbsDeltaFmax(u4,theta1)*exp(LogDFmax(u4)/2))
intval ans =
[  -0.16168616388582,  -0.16168616388578]
intval ans =
[   0.52448591012420,   0.52448591012422]
intval ans =
[   0.62831853071795,   0.62831853071796]
intval ans =
[  -0.45300852954273,  -0.45300852954272]
intval ans =
[  -0.47123889803847,  -0.47123889803846]
intval ans =
[   0.22756989531066,   0.22756989531067]
intval ans =
[   0.08396094459145,   0.08396094459146]
>>
>>
>> infsup(1,1)*tan(1.245)
infsup(1,1)*sqrt(1+8^2)/0.083

u6=infsup(10.7,10.7);
b1/u6+Q2max(u6)
(22-b0)*cos(theta1)-u6
LogDQmax(u6)
LogDvarphimax(u6)
intval ans =
[   2.96002722022931,   2.96002722022932]
```

```
intval ans =
[  97.13563552166927,  97.13563552166929]
intval ans =
[   2.30676792952983,   2.30676792952984]
intval ans =
[   2.38826771210230,   2.38826771210232]
intval ans =
[   0.24337117780356,   0.24337117780357]
intval ans =
[   0.01016458475250,   0.01016458475251]
>>
>>
>> t0=6.5*sqrt(2)-cp;

tt0=ones(1,12);
for k = 1:8
  tt0(k)=0.5*k;
end

for k = 9:10
  tt0(k)=0.8+0.4*k;
end

for k = 11:12
  tt0(k)=2.8+0.2*k;
end


-3*pi/infsup(4,4)
vartheta(0,tt0(1))
for k = 1:11
  vartheta(tt0(k),tt0(k+1))
end

pi/infsup(2,2)
vartheta(tt0(1),0)
for k = 1:11
  vartheta(tt0(k+1),tt0(k))
end
intval ans =
[  -2.35619449019235,  -2.35619449019234]
intval ans =
[  -1.23514999751268,  -1.23514999751099]
intval ans =
[  -1.49606541821700,  -1.49606541821530]
intval ans =
[  -1.71508257093239,  -1.71508257093061]
intval ans =
[  -1.89431544511375,  -1.89431544511195]
intval ans =
[  -2.03857421441208,  -2.03857421441032]
intval ans =
[  -2.15388764171335,  -2.15388764171166]
intval ans =
[  -2.24631547239388,  -2.24631547239230]
intval ans =
[  -2.32118907111286,  -2.32118907111138]
intval ans =
[  -2.28246752262180,  -2.28246752262044]
```

```
intval ans =
[  -2.33491721226844,  -2.33491721226715]
intval ans =
[  -2.21302433840200,  -2.21302433840079]
intval ans =
[  -2.24272766053250,  -2.24272766053133]
intval ans =
[   1.57079632679489,   1.57079632679490]
intval ans =
[   0.93455688536896,   0.93455688537066]
intval ans =
[   0.60229702096722,   0.60229702096900]
intval ans =
[   0.25233978854204,   0.25233978854384]
intval ans =
[  -0.09776367079340,  -0.09776367079164]
intval ans =
[  -0.43125703520784,  -0.43125703520616]
intval ans =
[  -0.73604218090996,  -0.73604218090838]
intval ans =
[  -1.00606295846654,  -1.00606295846507]
intval ans =
[  -1.24038286436472,  -1.24038286436335]
intval ans =
[  -1.51909522077555,  -1.51909522077427]
intval ans =
[  -1.65060363785092,  -1.65060363784970]
intval ans =
[  -1.89754846118753,  -1.89754846118635]
intval ans =
[  -1.94356760599094,  -1.94356760598980]
>>
>>
>> atan(t0/cp)
pi/infsup(4,4)
atan((t0-imag(omega))/(cp-real(omega)))
atan((t0+imag(omega))/(cp-real(omega)))
intval ans =
[   0.89859046656085,   0.89859046656087]
intval ans =
[   0.78539816339744,   0.78539816339745]
intval ans =
[   0.90827852884623,   0.90827852884624]
intval ans =
[   1.04428623924510,   1.04428623924511]
>>
>>
>> (6.5+b0)*cos(pi/4)+2^4*a11/(5^5*(6.5-1)^2)*cos(pi/4)+Q3max(6.5)
cv*cos(pi/4)
intval ans =
[  10.73024502229519,  10.73024502229521]
intval ans =
[  11.98329510308299,  11.98329510308301]
>>
>>
>> cp+c00-c01max-varphi1max(cp)
6.5+c00*cos(pi/4)-c01max-varphi1max(6.5)
intval ans =
```

```
[   1.70318641690299,   1.70318641690300]
intval ans =
[   4.22340128593619,   4.22340128593620]
>>
>>
>> xx1=ones(1,16);
for k = 1:16
  xx1(k)=0.04*k;
end

-5*pi/infsup(4,4)

theta21(xx1(1))+(theta22(xx1(1))+4*pi)-pi/2-(theta24(0)+4*pi)-(theta25(0)+4*pi)

for k = 1:15
  Theta2(xx1(k),xx1(k+1))-4*pi
end

theta21(real(omega))+theta22(real(omega))-theta23(xx1(16))-theta24(xx1(16))-theta25↙
(xx1(16))-4*pi

theta21(0)+theta22(0)-theta23(xx1(1))-theta24(xx1(1))-theta25(xx1(1))-4*pi

for k = 1:15
  Theta2(xx1(k+1),xx1(k))-4*pi
end

theta21(0.64)+(theta22(0.64)+4*pi)-theta23(real(omega))-2*pi-2*pi
-3*pi/infsup(4,4)
intval ans =
[  -3.92699081698725,  -3.92699081698724]
intval ans =
[  -2.84850446696732,  -2.84850446696729]
intval ans =
[  -2.93209795739691,  -2.93209795739687]
intval ans =
[  -3.01280662533490,  -3.01280662533485]
intval ans =
[  -3.08985845339689,  -3.08985845339685]
intval ans =
[  -3.16236030243339,  -3.16236030243335]
intval ans =
[  -3.22927206957840,  -3.22927206957837]
intval ans =
[  -3.28937247414529,  -3.28937247414526]
intval ans =
[  -3.34121331505448,  -3.34121331505444]
intval ans =
[  -3.38305783736550,  -3.38305783736547]
intval ans =
[  -3.41279722260158,  -3.41279722260155]
intval ans =
[  -3.42783712893471,  -3.42783712893468]
intval ans =
[  -3.42494377609214,  -3.42494377609210]
intval ans =
[  -3.40003694442663,  -3.40003694442659]
intval ans =
[  -3.34791750071525,  -3.34791750071521]
```

```
intval ans =
[  -3.26192528180115,  -3.26192528180111]
intval ans =
[  -3.13355327301950,  -3.13355327301946]
intval ans =
[  -2.84850331003140,  -2.84850331003134]
intval ans =
[  -2.54968710012177,  -2.54968710012174]
intval ans =
[  -2.62256715206858,  -2.62256715206854]
intval ans =
[  -2.69188144691537,  -2.69188144691534]
intval ans =
[  -2.75673642745087,  -2.75673642745083]
intval ans =
[  -2.81608473685361,  -2.81608473685357]
intval ans =
[  -2.86868973713925,  -2.86868973713921]
intval ans =
[  -2.91307842679135,  -2.91307842679131]
intval ans =
[  -2.94747839896368,  -2.94747839896364]
intval ans =
[  -2.96973285591276,  -2.96973285591273]
intval ans =
[  -2.97718560330167,  -2.97718560330163]
intval ans =
[  -2.96652551365350,  -2.96652551365346]
intval ans =
[  -2.93357782125675,  -2.93357782125671]
intval ans =
[  -2.87302985614146,  -2.87302985614141]
intval ans =
[  -2.77808705256084,  -2.77808705256080]
intval ans =
[  -2.64008519460844,  -2.64008519460840]
intval ans =
[  -2.44816840193517,  -2.44816840193513]
intval ans =
[  -2.40422785175702,  -2.40422785175699]
intval ans =
[  -2.35619449019235,  -2.35619449019234]
>>
>>
>> Theta2(-1,-0.975)-5*pi
Theta2(-0.975,-0.95)-5*pi
Theta2(-0.95,-0.925)-5*pi
Theta2(-0.925,-0.9)-5*pi

6*pi/2+theta22(-1)-theta23(-0.975)-theta24(-0.975)-theta25(-0.975)-5*pi
Theta2(-0.95,-0.975)-5*pi
Theta2(-0.925,-0.95)-5*pi
Theta2(-0.9,-0.925)-5*pi
intval ans =
[  -0.81321631031848,  -0.81321631031845]
intval ans =
[  -0.85148437015118,  -0.85148437015115]
intval ans =
[  -0.89039122686946,  -0.89039122686943]
```

```
intval ans =
[  -0.92994285272024,  -0.92994285272021]
intval ans =
[  -0.72923865212319,  -0.72923865212317]
intval ans =
[  -0.76592178317752,  -0.76592178317748]
intval ans =
[  -0.80320746989739,  -0.80320746989736]
intval ans =
[  -0.84110092166078,  -0.84110092166075]
>>
>>
>> Xi2(-0.975,-1)*cos(6*pi/2+theta22(-1)-theta23(-0.975)-theta24(-0.975)-theta25↙
(-0.975)-5*pi)
h2(-0.975,-0.95)
h2(-0.95,-0.925)
h2(-0.925,-0.9)

-Xi2(-0.975,-1)-(1.7-4.2*sqrt(2))
-Xi2(-0.95,-0.975)-(1.7-4.2*sqrt(2))
-Xi2(-0.925,-0.95)-(1.7-4.2*sqrt(2))
-Xi2(-0.9,-0.925) -(1.7-4.2*sqrt(2))
intval ans =
[   1.65018873952891,   1.65018873952897]
intval ans =
[   1.60327690654197,   1.60327690654204]
intval ans =
[   1.55375985879196,   1.55375985879202]
intval ans =
[   1.50149177242773,   1.50149177242779]
intval ans =
[   2.02670409461156,   2.02670409461160]
intval ans =
[   2.01522112351874,   2.01522112351878]
intval ans =
[   2.00214675187856,   2.00214675187860]
intval ans =
[   1.98737808845005,   1.98737808845010]
>>
>>
>> xx2=ones(1,22);
for k = 4:22
  xx2(k)=-1.1+0.05*k;
end

for k = 4:21
  Theta2(xx2(k),xx2(k+1))-5*pi
end

for k = 4:20
  Theta2(xx2(k+1),xx2(k))-5*pi
end

theta21kmax=theta21(-0.05)+(theta22(-0.05)+4*pi)-pi/2-(theta24(0)+4*pi)-(theta25(0)↙
+4*pi)
intval ans =
[  -1.03643500329737,  -1.03643500329734]
intval ans =
[  -1.12124355297376,  -1.12124355297372]
```

```
intval ans =
[  -1.20878509868440,  -1.20878509868437]
intval ans =
[  -1.29907363546060,  -1.29907363546057]
intval ans =
[  -1.39210526544360,  -1.39210526544357]
intval ans =
[  -1.48785462422675,  -1.48785462422672]
intval ans =
[  -1.58627082352583,  -1.58627082352580]
intval ans =
[  -1.68727286617838,  -1.68727286617835]
intval ans =
[  -1.79074448635401,  -1.79074448635398]
intval ans =
[  -1.89652836242111,  -1.89652836242107]
intval ans =
[  -2.00441964023564,  -2.00441964023561]
intval ans =
[  -2.11415868773827,  -2.11415868773823]
intval ans =
[  -2.22542297323602,  -2.22542297323599]
intval ans =
[  -2.33781791309277,  -2.33781791309274]
intval ans =
[  -2.45086646028052,  -2.45086646028049]
intval ans =
[  -2.56399708963122,  -2.56399708963119]
intval ans =
[  -2.67652965877806,  -2.67652965877803]
intval ans =
[  -2.78765835665317,  -2.78765835665314]
intval ans =
[  -0.85362474912925,  -0.85362474912922]
intval ans =
[  -0.93133719629989,  -0.93133719629986]
intval ans =
[  -1.01145847952171,  -1.01145847952167]
intval ans =
[  -1.09398973503134,  -1.09398973503131]
intval ans =
[  -1.17891391140139,  -1.17891391140136]
intval ans =
[  -1.26619208549129,  -1.26619208549126]
intval ans =
[  -1.35575922302910,  -1.35575922302907]
intval ans =
[  -1.44751931257862,  -1.44751931257859]
intval ans =
[  -1.54133979052806,  -1.54133979052802]
intval ans =
[  -1.63704515919203,  -1.63704515919200]
intval ans =
[  -1.73440967757299,  -1.73440967757296]
intval ans =
[  -1.83314897086097,  -1.83314897086094]
intval ans =
[  -1.93291035447247,  -1.93291035447244]
intval ans =
```

```
[  -2.03326159255830,  -2.03326159255826]
intval ans =
[  -2.13367769641782,  -2.13367769641779]
intval ans =
[  -2.23352519582512,  -2.23352519582509]
intval ans =
[  -2.33204305718539,  -2.33204305718536]
intval theta21kmax =
[  -2.42831903377874,  -2.42831903377871]
>>
>>
>> for k = 4:20
  h2(xx2(k),xx2(k+1))
end

Xi2(0,-0.05)*cos(theta21kmax)

for k = 4:20
  vv2(xx2(k),xx2(k+1))
end

Xi2(0,-0.05)*sin(theta21kmax)-(Xi2(0,-0.05)*cos(theta21kmax)-4.2*sqrt(2))
intval ans =
[   1.65345995750203,   1.65345995750211]
intval ans =
[   1.53073671626427,   1.53073671626435]
intval ans =
[   1.39278744559756,   1.39278744559765]
intval ans =
[   1.23755131514378,   1.23755131514387]
intval ans =
[   1.06259011741628,   1.06259011741638]
intval ans =
[   0.86500021668170,   0.86500021668178]
intval ans =
[   0.64129926927424,   0.64129926927432]
intval ans =
[   0.38727907424337,   0.38727907424346]
intval ans =
[   0.09781244970305,   0.09781244970314]
intval ans =
[  -0.23340302966709,  -0.23340302966700]
intval ans =
[  -0.61418908847202,  -0.61418908847191]
intval ans =
[  -1.05435449478017,  -1.05435449478004]
intval ans =
[  -1.56633949995065,  -1.56633949995051]
intval ans =
[  -2.16611642273446,  -2.16611642273431]
intval ans =
[  -2.87446731178331,  -2.87446731178313]
intval ans =
[  -3.71882573206009,  -3.71882573205988]
intval ans =
[  -4.73597673421545,  -4.73597673421523]
intval ans =
[  -5.97608494757809,  -5.97608494757781]
intval ans =
```

```
[   2.39023277001314,   2.39023277001329]
intval ans =
[   2.35069805719454,   2.35069805719468]
intval ans =
[   2.32210716219904,   2.32210716219919]
intval ans =
[   2.30638408279710,   2.30638408279726]
intval ans =
[   2.30585008147688,   2.30585008147705]
intval ans =
[   2.32332137529099,   2.32332137529114]
intval ans =
[   2.36223510692243,   2.36223510692258]
intval ans =
[   2.42681307992009,   2.42681307992024]
intval ans =
[   2.52227636260337,   2.52227636260352]
intval ans =
[   2.65512903540611,   2.65512903540627]
intval ans =
[   2.83353680329148,   2.83353680329168]
intval ans =
[   3.06783701317037,   3.06783701317061]
intval ans =
[   3.37123245859760,   3.37123245859787]
intval ans =
[   3.76074471503192,   3.76074471503221]
intval ans =
[   4.25853737261682,   4.25853737261717]
intval ans =
[   4.89377094885166,   4.89377094885205]
intval ans =
[   5.70522734525410,   5.70522734525452]
intval ans =
[   6.74505279659459,   6.74505279659511]
>>
>>
>> xx3=ones(1,40);
for k = 1:20
  xx3(k)=-1-0.04*k;
end

for k = 21:29
  xx3(k)=2.2-0.2*k;
end

for k = 30:40
  xx3(k)=8-0.4*k;
end

theta31kmin=6*pi/2+(theta32(-1)+4*pi)-(theta33(-1.04)+pi)-(theta34(-1.04)+4*pi)-↙
(theta35(-1.04)+4*pi)

for k = 1:7
  Theta3(xx3(k+1),xx3(k))
end

theta31kmax=Theta3(-1,-1.04)
```

```
for k = 1:7
  Theta3(xx3(k),xx3(k+1))
end
intval theta31kmin =
[  -0.84834442345356,  -0.84834442345352]
intval ans =
[  -0.82020109973416,  -0.82020109973413]
intval ans =
[  -0.79129758969616,  -0.79129758969613]
intval ans =
[  -0.76167371111086,  -0.76167371111083]
intval ans =
[  -0.73136962579683,  -0.73136962579680]
intval ans =
[  -0.70042571039302,  -0.70042571039298]
intval ans =
[  -0.66888243144226,  -0.66888243144223]
intval ans =
[  -0.63678022517161,  -0.63678022517158]
intval theta31kmax =
[  -0.63010482164984,  -0.63010482164981]
intval ans =
[  -0.60380125453010,  -0.60380125453007]
intval ans =
[  -0.57675908101592,  -0.57675908101589]
intval ans =
[  -0.54901691635535,  -0.54901691635532]
intval ans =
[  -0.52061372750860,  -0.52061372750857]
intval ans =
[  -0.49158870114494,  -0.49158870114491]
intval ans =
[  -0.46198111644396,  -0.46198111644393]
intval ans =
[  -0.43183022309108,  -0.43183022309105]
>>
>>
>> 4.2*sqrt(infsup(2,2))-1.7
Xi3(-1,-1.04)

for k = 1:7
  Xi3(xx3(k),xx3(k+1))
end
intval ans =
[   4.23969696196699,   4.23969696196701]
intval ans =
[   2.06265693262300,   2.06265693262304]
intval ans =
[   2.00015852864594,   2.00015852864599]
intval ans =
[   1.94109148567547,   1.94109148567551]
intval ans =
[   1.88530037777485,   1.88530037777489]
intval ans =
[   1.83263470062486,   1.83263470062490]
intval ans =
[   1.78294911664355,   1.78294911664359]
intval ans =
[   1.73610362864887,   1.73610362864891]
```

```
intval ans =
[   1.69196369061738,   1.69196369061741]
>>
>>
>> Xi3(-1,-1.04)*cos(Theta3(-1,-1.04))

for k = 1:7
  Xi3(xx3(k),xx3(k+1))*cos(Theta3(xx3(k),xx3(k+1)))
end

for k = 8:28
  Xi3(xx3(k),xx3(k+1))
end
intval ans =
[   1.66655615297619,   1.66655615297626]
intval ans =
[   1.64649711307029,   1.64649711307035]
intval ans =
[   1.62708956963518,   1.62708956963525]
intval ans =
[   1.60823277934598,   1.60823277934604]
intval ans =
[   1.58983638292387,   1.58983638292392]
intval ans =
[   1.57181952378062,   1.57181952378068]
intval ans =
[   1.55411001508004,   1.55411001508009]
intval ans =
[   1.53664355762830,   1.53664355762835]
intval ans =
[   1.65040026355425,   1.65040026355428]
intval ans =
[   1.61128982391474,   1.61128982391477]
intval ans =
[   1.57451433142330,   1.57451433142334]
intval ans =
[   1.53996116253884,   1.53996116253887]
intval ans =
[   1.50752301522537,   1.50752301522540]
intval ans =
[   1.47709779011672,   1.47709779011675]
intval ans =
[   1.44858845261738,   1.44858845261742]
intval ans =
[   1.42190287996630,   1.42190287996633]
intval ans =
[   1.39695369680779,   1.39695369680782]
intval ans =
[   1.37365810236728,   1.37365810236731]
intval ans =
[   1.35193769191883,   1.35193769191886]
intval ans =
[   1.33171827485729,   1.33171827485732]
intval ans =
[   1.62545992961129,   1.62545992961132]
intval ans =
[   1.55756088592176,   1.55756088592179]
intval ans =
[   1.51751956353777,   1.51751956353781]
```

```
intval ans =
[   1.50038672415246,   1.50038672415249]
intval ans =
[   1.50235818483395,   1.50235818483398]
intval ans =
[   1.52049929292683,   1.52049929292686]
intval ans =
[   1.55252776179398,   1.55252776179401]
intval ans =
[   1.59664804201207,   1.59664804201210]
intval ans =
[   1.65142753022827,   1.65142753022830]
>>
>>
>> for k = 29:39
  Theta3(xx3(k+1),xx3(k))
end

for k = 29:39
  Theta3(xx3(k),xx3(k+1))
end
intval ans =
[   0.88414108885983,   0.88414108885986]
intval ans =
[   1.13885871076736,   1.13885871076738]
intval ans =
[   1.35650174669510,   1.35650174669512]
intval ans =
[   1.54232999766212,   1.54232999766214]
intval ans =
[   1.70127796365123,   1.70127796365126]
intval ans =
[   1.83769221201597,   1.83769221201599]
intval ans =
[   1.95527211903493,   1.95527211903496]
intval ans =
[   2.05710697032961,   2.05710697032963]
intval ans =
[   2.14575083366281,   2.14575083366283]
intval ans =
[   2.22330576937272,   2.22330576937274]
intval ans =
[   2.29150003468029,   2.29150003468031]
intval ans =
[   1.82130519334237,   1.82130519334239]
intval ans =
[   1.96028602283541,   1.96028602283544]
intval ans =
[   2.07922612437740,   2.07922612437742]
intval ans =
[   2.18103712867743,   2.18103712867745]
intval ans =
[   2.26840588675817,   2.26840588675819]
intval ans =
[   2.34367598685467,   2.34367598685470]
intval ans =
[   2.40883035776697,   2.40883035776699]
intval ans =
[   2.46551925652007,   2.46551925652009]
```

```
intval ans =
[   2.51510455507618,   2.51510455507620]
intval ans =
[   2.55870626734773,   2.55870626734775]
intval ans =
[   2.59724522264411,   2.59724522264413]
>>
>>
>> for k = 29:39
  h3(xx3(k),xx3(k+1))
end

for k = 29:39
  vv3(xx3(k),xx3(k+1))
end
intval ans =
[   1.52069658865453,   1.52069658865459]
intval ans =
[   1.08580721500490,   1.08580721500496]
intval ans =
[   0.59925545598504,   0.59925545598509]
intval ans =
[   0.08726881388892,   0.08726881388897]
intval ans =
[  -0.43380800543922,  -0.43380800543916]
intval ans =
[  -0.95440194338462,  -0.95440194338455]
intval ans =
[  -1.46929704111150,  -1.46929704111143]
intval ans =
[  -1.97595758570020,  -1.97595758570013]
intval ans =
[  -2.47343416325068,  -2.47343416325061]
intval ans =
[  -2.96167747091833,  -2.96167747091826]
intval ans =
[  -3.44111977232795,  -3.44111977232787]
intval ans =
[  -2.56387792776937,  -2.56387792776926]
intval ans =
[  -2.49839745439003,  -2.49839745438993]
intval ans =
[  -2.58696909950825,  -2.58696909950816]
intval ans =
[  -2.78757071689683,  -2.78757071689675]
intval ans =
[  -3.06772730906189,  -3.06772730906180]
intval ans =
[  -3.40348000918749,  -3.40348000918738]
intval ans =
[  -3.77761994787189,  -3.77761994787176]
intval ans =
[  -4.17797172003901,  -4.17797172003889]
intval ans =
[  -4.59599057196808,  -4.59599057196795]
intval ans =
[  -5.02570400893342,  -5.02570400893328]
intval ans =
[  -5.46294574897112,  -5.46294574897098]
```

```
>>
>>
>> 2*Q2max(8)-8-1.6+b0
intval ans =
[  -0.93963548665499,  -0.93963548665497]
>>
>>
>> x42m=(2*real(a4)*imag(a4)*imag(omega)+(real(a4)^2-imag(a4)^2)*real(omega))/(real ↙
(a4)^2+imag(a4)^2)
y4m(x42m)
x40m=-0.22-epsilon4
x41m=-1;

s41=-((1+real(a4))*imag(a4)+epsilon4*sqrt((1+real(a4))^2+(imag(a4))^2-epsilon4^2))/ ↙
(epsilon4^2-(1+real(a4))^2)
x41tilde=(real(a4)+imag(a4)*s41-s41^2)/(1+s41^2)

s42=-((1-real(a4))*imag(a4)+epsilon4*sqrt((1-real(a4))^2+(imag(a4))^2-epsilon4^2))/ ↙
((1-real(a4))^2- epsilon4^2);
x42tilde=(real(a4)+imag(a4)*s42+s42^2)/(1+s42^2)
intval x42m =
[  -0.97422037135259,  -0.97422037135258]
intval ans =
[   0.22559846639911,   0.22559846639913]
intval x40m =
[  -1.10572972907885,  -1.10572972907884]
intval s41 =
[  -5.81045211005577,  -5.81045211005563]
intval x41tilde =
[  -1.09289661103676,  -1.09289661103666]
intval x42tilde =
[   0.60514061962698,   0.60514061962701]
>>
>>
>> x401m=-1.096;
theta41m(x401m)+theta42m(x401m)-atan(imag(a4)/x40m)-4*atan((imag(a4)-imag(omega))/ ↙
(x40m-real(omega)))-4*atan((imag(a4)+imag(omega))/(x40m-real(omega)))+pi+2*pi

6*atan(imag(a4)/(x40m+1))+ 4*atan(imag(a4)/(x40m-1))-theta43m(x401m)-theta44m(x401m)- ↙
theta45m(x401m)+pi+2*pi


6*atan(s41)+theta42m(-1)-theta43m(x401m)-theta44m(x401m)- theta45m(x401m)+pi+2*pi
6*(-pi/2)+theta42m(x401m)-theta43m(-1)-theta44m(-1)- theta45m(-1)+pi+2*pi

x411m=-1.04;
Theta41mm(x41tilde,x411m)
Theta41mm(x411m,x41tilde)
Theta41mm(x411m,-1)
Theta41mm(-1,x411m)
intval ans =
[   3.13418076253030,   3.13418076253037]
intval ans =
[   2.24502382998528,   2.24502382998538]
intval ans =
[   3.08452471530704,   3.08452471530715]
intval ans =
[   0.30005875595706,   0.30005875595713]
intval ans =
```

```
[   3.08601065478581,   3.08601065478849]
intval ans =
[   1.14612314292547,   1.14612314292658]
intval ans =
[   2.15574703263479,   2.15574703263486]
intval ans =
[   0.56883884835599,   0.56883884835605]
>>
>>
>> x40p=-0.22-epsilon4;
x41p=-1;
x42p=0;
x43p=real(omega);

xx40=ones(1,14);
for k = 1:6
  xx40(k)=-1.11+0.005*k;
end

for k = 7:14
  xx40(k)=-1.14+0.01*k;
end

6*atan(imag(a4)/(x40p+1))+ 4*atan(imag(a4)/(x40p-1))-theta43p(xx40(1))-theta44p(xx40↙
(1))-theta45p(xx40(1))+pi+2*pi

for k = 1:13
  Theta4p(xx40(k+1),xx40(k))+pi+2*pi
end

theta41p(xx40(1))+theta42p(xx40(1))- atan(imag(a4)/ x40p)- 4*atan((imag(a4)-imag↙
(omega))/(x40p-real(omega)))-4*atan((imag(a4)+imag(omega))/(x40p-real(omega)))↙
+pi+2*pi

for k = 1:12
  Theta4p(xx40(k),xx40(k+1))+pi+2*pi
end

-3*pi+theta42p(xx40(14))-theta43p(xx40(13))-theta44p(xx40(13))-theta45p(xx40(13))↙
+pi+2*pi
intval ans =
[   2.95602846971200,   2.95602846971224]
intval ans =
[   3.12250275932111,   3.12250275932133]
intval ans =
[   3.06430416127335,   3.06430416127347]
intval ans =
[   3.05706200297872,   3.05706200297882]
intval ans =
[   3.05666806606283,   3.05666806606293]
intval ans =
[   3.05814196421497,   3.05814196421505]
intval ans =
[   3.14043964842154,   3.14043964842161]
intval ans =
[   3.13493893700022,   3.13493893700029]
intval ans =
[   3.13166439699197,   3.13166439699203]
intval ans =
```

```
[   3.12929480824936,   3.12929480824942]
intval ans =
[   3.12730207169822,   3.12730207169829]
intval ans =
[   3.12544939921511,   3.12544939921516]
intval ans =
[   3.12362373165284,   3.12362373165289]
intval ans =
[   3.12176981187097,   3.12176981187103]
intval ans =
[   2.68999698327271,   2.68999698327287]
intval ans =
[   2.62503504403590,   2.62503504403616]
intval ans =
[   2.77026140692447,   2.77026140692459]
intval ans =
[   2.82279292542442,   2.82279292542452]
intval ans =
[   2.85491144304640,   2.85491144304650]
intval ans =
[   2.87767715231728,   2.87767715231736]
intval ans =
[   2.82199927201670,   2.82199927201677]
intval ans =
[   2.85478890237103,   2.85478890237110]
intval ans =
[   2.87783640382414,   2.87783640382420]
intval ans =
[   2.89501464262160,   2.89501464262166]
intval ans =
[   2.90828771252621,   2.90828771252627]
intval ans =
[   2.91878694957361,   2.91878694957366]
intval ans =
[   2.92722234697428,   2.92722234697434]
intval ans =
[   2.93406794464782,   2.93406794464787]
>>
>>
>> xx41=ones(1,50);
for k = 1:20
  xx41(k)=-1+0.01*k;
end

for k = 21:40
  xx41(k)=-1.2+0.02*k;
end

for k = 41:50
  xx41(k)=-2+0.04*k;
end

3*pi+theta42p(-1)-theta43p(xx41(1))-theta44p(xx41(1))-theta45p(xx41(1))-5*pi+2*pi

for k = 1:48
  Theta4p(xx41(k+1),xx41(k))-5*pi+2*pi
end

theta41p(-0.04)+theta42p(-0.04)+pi/2-theta44p(0)-theta45p(0)-5*pi+2*pi
```

```
Theta4p(-1,-0.99)-5*pi+2*pi
for k = 1:49
  Theta4p(xx41(k),xx41(k+1))-5*pi+2*pi
end
intval ans =
[   3.11986117912591,   3.11986117912595]
intval ans =
[   3.11788633556230,   3.11788633556235]
intval ans =
[   3.11584174221402,   3.11584174221407]
intval ans =
[   3.11372811131942,   3.11372811131947]
intval ans =
[   3.11154837892328,   3.11154837892333]
intval ans =
[   3.10930657183739,   3.10930657183743]
intval ans =
[   3.10700716720649,   3.10700716720654]
intval ans =
[   3.10465473072826,   3.10465473072830]
intval ans =
[   3.10225371567140,   3.10225371567144]
intval ans =
[   3.09980835593517,   3.09980835593521]
intval ans =
[   3.09732261446689,   3.09732261446694]
intval ans =
[   3.09480016421196,   3.09480016421201]
intval ans =
[   3.09224438794186,   3.09224438794191]
intval ans =
[   3.08965838871683,   3.08965838871687]
intval ans =
[   3.08704500598473,   3.08704500598477]
intval ans =
[   3.08440683429107,   3.08440683429111]
intval ans =
[   3.08174624278608,   3.08174624278614]
intval ans =
[   3.07906539446430,   3.07906539446434]
intval ans =
[   3.07636626453601,   3.07636626453605]
intval ans =
[   3.07365065761874,   3.07365065761878]
intval ans =
[   3.12847607665878,   3.12847607665882]
intval ans =
[   3.12166149405137,   3.12166149405141]
intval ans =
[   3.11493136153076,   3.11493136153080]
intval ans =
[   3.10827808085842,   3.10827808085846]
intval ans =
[   3.10169563750736,   3.10169563750742]
intval ans =
[   3.09517920304528,   3.09517920304532]
intval ans =
[   3.08872485437692,   3.08872485437696]
```

```
intval ans =
[   3.08232937241501,   3.08232937241505]
intval ans =
[   3.07599009573442,   3.07599009573447]
intval ans =
[   3.06970481293707,   3.06970481293712]
intval ans =
[   3.06347168270570,   3.06347168270574]
intval ans =
[   3.05728917396493,   3.05728917396498]
intval ans =
[   3.05115602086038,   3.05115602086042]
intval ans =
[   3.04507118882017,   3.04507118882021]
intval ans =
[   3.03903384903130,   3.03903384903134]
intval ans =
[   3.03304335940897,   3.03304335940902]
intval ans =
[   3.02709925066399,   3.02709925066403]
intval ans =
[   3.02120121645185,   3.02120121645189]
intval ans =
[   3.01534910686221,   3.01534910686225]
intval ans =
[   3.00954292471053,   3.00954292471057]
intval ans =
[   3.09328112670877,   3.09328112670881]
intval ans =
[   3.08117635902017,   3.08117635902021]
intval ans =
[   3.06944244840975,   3.06944244840979]
intval ans =
[   3.05807738419397,   3.05807738419401]
intval ans =
[   3.04708542930813,   3.04708542930817]
intval ans =
[   3.03647728050110,   3.03647728050115]
intval ans =
[   3.02627054171922,   3.02627054171926]
intval ans =
[   3.01649053899727,   3.01649053899731]
intval ans =
[   3.00717153884645,   3.00717153884649]
intval ans =
[   2.99835847400567,   2.99835847400571]
intval ans =
[   2.93965545333875,   2.93965545333880]
intval ans =
[   2.94422569795279,   2.94422569795284]
intval ans =
[   2.94795878924198,   2.94795878924203]
intval ans =
[   2.95099276525820,   2.95099276525825]
intval ans =
[   2.95343560981700,   2.95343560981705]
intval ans =
[   2.95537328095546,   2.95537328095551]
intval ans =
```

```
[   2.95687523768406,   2.95687523768411]
intval ans =
[   2.95799834399347,   2.95799834399351]
intval ans =
[   2.95878968900754,   2.95878968900759]
intval ans =
[   2.95928866453646,   2.95928866453651]
intval ans =
[   2.95952852232070,   2.95952852232074]
intval ans =
[   2.95953755941316,   2.95953755941320]
intval ans =
[   2.95934003304969,   2.95934003304973]
intval ans =
[   2.95895687558391,   2.95895687558395]
intval ans =
[   2.95840625951457,   2.95840625951461]
intval ans =
[   2.95770404864307,   2.95770404864312]
intval ans =
[   2.95686416170295,   2.95686416170300]
intval ans =
[   2.95589886797485,   2.95589886797489]
intval ans =
[   2.95481902952022,   2.95481902952026]
intval ans =
[   2.95363430113134,   2.95363430113138]
intval ans =
[   2.89271640451390,   2.89271640451394]
intval ans =
[   2.89112470764972,   2.89112470764977]
intval ans =
[   2.88911657168124,   2.88911657168128]
intval ans =
[   2.88674943226509,   2.88674943226513]
intval ans =
[   2.88407065636239,   2.88407065636244]
intval ans =
[   2.88111967926937,   2.88111967926942]
intval ans =
[   2.87792961314314,   2.87792961314318]
intval ans =
[   2.87452847524827,   2.87452847524831]
intval ans =
[   2.87094013805774,   2.87094013805778]
intval ans =
[   2.86718507288579,   2.86718507288583]
intval ans =
[   2.86328093819814,   2.86328093819819]
intval ans =
[   2.85924304964833,   2.85924304964837]
intval ans =
[   2.85508475904998,   2.85508475905002]
intval ans =
[   2.85081776252321,   2.85081776252325]
intval ans =
[   2.84645235304394,   2.84645235304398]
intval ans =
[   2.84199762898081,   2.84199762898085]
```

```
intval ans =
[   2.83746166752073,   2.83746166752077]
intval ans =
[   2.83285166988795,   2.83285166988799]
intval ans =
[   2.82817408376041,   2.82817408376045]
intval ans =
[   2.82343470714938,   2.82343470714942]
intval ans =
[   2.72385901379046,   2.72385901379050]
intval ans =
[   2.71484425565775,   2.71484425565779]
intval ans =
[   2.70548321868465,   2.70548321868469]
intval ans =
[   2.69581310372950,   2.69581310372954]
intval ans =
[   2.68586457897368,   2.68586457897374]
intval ans =
[   2.67566318425342,   2.67566318425347]
intval ans =
[   2.66523044991772,   2.66523044991776]
intval ans =
[   2.65458481702438,   2.65458481702442]
intval ans =
[   2.64374242541515,   2.64374242541519]
intval ans =
[   2.63271782418823,   2.63271782418828]
>>
>>
>> xx42=ones(1,26);
for k = 1:11
  xx42(k)=0.05*k;
end

xx42(12)=0.575;

Theta4p(xx42(1),0)-4*pi+2*pi

for k = 1:11
  Theta4p(xx42(k+1),xx42(k))-4*pi+2*pi
end

Theta4p(x42tilde,xx42(12))-4*pi+2*pi

theta41p(xx42(1))+theta42p(xx42(1))-pi/2-theta44p(0)-theta45p(0)-4*pi+2*pi

for k = 1:11
  Theta4p(xx42(k),xx42(k+1))-4*pi+2*pi
end

Theta4p(xx42(12), x42tilde)-4*pi+2*pi
intval ans =
[   3.03527624873705,   3.03527624873709]
intval ans =
[   3.02660631808298,   3.02660631808303]
intval ans =
[   3.01933184285401,   3.01933184285405]
intval ans =
```

```
[   3.01375857373411,   3.01375857373415]
intval ans =
[   3.01031557649248,   3.01031557649252]
intval ans =
[   3.00961949980789,   3.00961949980794]
intval ans =
[   3.01258636427324,   3.01258636427329]
intval ans =
[   3.02063910852584,   3.02063910852589]
intval ans =
[   3.03612749928545,   3.03612749928550]
intval ans =
[   3.06328439906697,   3.06328439906703]
intval ans =
[   3.11082012409566,   3.11082012409573]
intval ans =
[   2.97273934363197,   2.97273934363204]
intval ans =
[   3.06668814627410,   3.06668814627481]
intval ans =
[   2.57399401823832,   2.57399401823836]
intval ans =
[   2.55914957137257,   2.55914957137261]
intval ans =
[   2.54391625362158,   2.54391625362162]
intval ans =
[   2.52829707756770,   2.52829707756776]
intval ans =
[   2.51229138260828,   2.51229138260832]
intval ans =
[   2.49589438180439,   2.49589438180444]
intval ans =
[   2.47909479093032,   2.47909479093037]
intval ans =
[   2.46186708696890,   2.46186708696895]
intval ans =
[   2.44414735164790,   2.44414735164795]
intval ans =
[   2.42575276241707,   2.42575276241713]
intval ans =
[   2.40606945006607,   2.40606945006613]
intval ans =
[   2.58257998158700,   2.58257998158707]
intval ans =
[   2.54325190923534,   2.54325190923578]
>>
>>
>> for k = 14:20
  xx42(k)=0.625+0.005*(k-14);
end

for k = 21:25
  xx42(k)=0.659+0.001*(k-21);
end

xx42(26)=0.6635;

Theta4phat(xx42(14),x42tilde)
for k = 14:25
```

```
  Theta4phat(xx42(k+1),xx42(k))
end

Theta4phat(x42tilde,xx42(14))
for k = 14:25
  Theta4phat(xx42(k),xx42(k+1))
end

theta41p(0.6635)+(theta42p(real(omega))+4*pi)-theta43p(real(omega))-4*(atan((imag↙
(omega)-0.69)/(real(omega)+0.22))+pi/2)-4*pi/2+2*pi

Theta4phat(0.6635,real(omega))
intval ans =
[   3.04864590390064,   3.04864590390094]
intval ans =
[   2.92979846524614,   2.92979846524624]
intval ans =
[   2.94922494426218,   2.94922494426229]
intval ans =
[   2.97236324979566,   2.97236324979578]
intval ans =
[   3.00068143898320,   3.00068143898333]
intval ans =
[   3.03677412369583,   3.03677412369599]
intval ans =
[   3.08597976952307,   3.08597976952327]
intval ans =
[   3.12419642405801,   3.12419642405831]
intval ans =
[   3.07398024341227,   3.07398024341263]
intval ans =
[   3.09047156551093,   3.09047156551136]
intval ans =
[   3.10994955149473,   3.10994955149532]
intval ans =
[   3.13399811255174,   3.13399811255287]
intval ans =
[   3.12985612238349,   3.12985612238602]
intval ans =
[   2.63734662730976,   2.63734662731064]
intval ans =
[   2.80952182742723,   2.80952182742733]
intval ans =
[   2.81927780562860,   2.81927780562871]
intval ans =
[   2.83017213431545,   2.83017213431556]
intval ans =
[   2.84232188635214,   2.84232188635226]
intval ans =
[   2.85571565761190,   2.85571565761205]
intval ans =
[   2.86976781100031,   2.86976781100048]
intval ans =
[   2.90699136228256,   2.90699136228278]
intval ans =
[   3.00929254529881,   3.00929254529912]
intval ans =
[   3.01941334051261,   3.01941334051297]
intval ans =
```

```
[   3.03033915558542,   3.03033915558586]
intval ans =
[   3.04203775808423,   3.04203775808484]
intval ans =
[   3.07710701815580,   3.07710701815695]
intval ans =
[   3.13310435455557,   3.13310435455579]
intval ans =
[   3.09398471246604,   3.09398471246864]
>>
>>
>> x51m=real(a5)+sqrt(1+(real(a5))^2-2*(real(a5)*real(omega)+imag(a5)*imag(omega)))
x52m=0.78+epsilon5
intval x51m =
[   1.28863164896621,   1.28863164896622]
intval x52m =
[   1.33027825173278,   1.33027825173279]
>>
>>
>> infsup(3,3)*pi/4

xx50=ones(1,12);
for k = 1:11
  xx50(k)=0.66+0.03*k;
end

xx50(12)=1;

for k = 1:10
  Theta5p(xx50(k),xx50(k+1))+4*pi
end

theta51p(1)+4*(-pi/2)-theta53p(xx50(11))-theta54p(xx50(11))-theta55p(xx50(11))+4*pi

for k = 1:11
  Theta5p(xx50(k+1),xx50(k))+4*pi
end
intval ans =
[   2.35619449019234,   2.35619449019235]
intval ans =
[   2.39152641523626,   2.39152641523674]
intval ans =
[   2.42360215407705,   2.42360215407730]
intval ans =
[   2.45094740795036,   2.45094740795053]
intval ans =
[   2.47375903624622,   2.47375903624636]
intval ans =
[   2.49216015800997,   2.49216015801008]
intval ans =
[   2.50620174545471,   2.50620174545480]
intval ans =
[   2.51586026123196,   2.51586026123205]
intval ans =
[   2.52103080942897,   2.52103080942905]
intval ans =
[   2.52151463955717,   2.52151463955725]
intval ans =
[   2.51699890505396,   2.51699890505403]
```

```
intval ans =
[   2.66361386315765,   2.66361386315772]
intval ans =
[   2.77182869339948,   2.77182869339972]
intval ans =
[   2.80697205644845,   2.80697205644862]
intval ans =
[   2.83822736489731,   2.83822736489744]
intval ans =
[   2.86584599820322,   2.86584599820334]
intval ans =
[   2.89002811640701,   2.89002811640711]
intval ans =
[   2.91092823563497,   2.91092823563506]
intval ans =
[   2.92865896605913,   2.92865896605922]
intval ans =
[   2.94329317499184,   2.94329317499192]
intval ans =
[   2.95486473453924,   2.95486473453932]
intval ans =
[   2.96336795294597,   2.96336795294604]
intval ans =
[   2.81568713772159,   2.81568713772166]
>>
>>
>> Theta5p(xx50(1),real(omega))+4*pi
theta51p(xx50(1))+theta52p(xx50(1))-theta53p(real(omega))-4*atan((0.78-real(omega))/↙
(imag(omega)-0.21))-4*pi/2+4*pi

infsup(3,3)*pi/4
intval ans =
[   2.71109759363439,   2.71109759363488]
intval ans =
[   2.38130456898430,   2.38130456898434]
intval ans =
[   2.35619449019234,   2.35619449019235]
>>
>>
>> xx51p=ones(1,23);
xx51p(1)=1.04;
xx51p(2)=1.08;

Theta5p(1,xx51p(1))
Theta5p(xx51p(1),xx51p(2))

theta51p(1)+4*pi/2-theta53p(xx51p(1))-theta54p(xx51p(1))-theta55p(xx51p(1))

Theta5p(xx51p(2),xx51p(1))
intval ans =
[   2.42084290987282,   2.42084290987289]
intval ans =
[   2.38854943594669,   2.38854943594676]
intval ans =
[   3.04826057324545,   3.04826057324551]
intval ans =
[   3.05274579760863,   3.05274579760869]
>>
>>
```

```
>> s51=imag(a5)/(real(a5)+1);
xtilde51=real(a5)+epsilon5/sqrt(1+s51^2)

s52=imag(a5)/real(a5);
xtilde52=real(a5)+epsilon5/sqrt(1+s52^2)

s53=imag(a5+omega)/real(a5-omega);
xtilde53=real(a5)+epsilon5/sqrt(1+s53^2)
intval xtilde51 =
[   1.32648819251098,   1.32648819251099]
intval xtilde52 =
[   1.31135735144438,   1.31135735144439]
intval xtilde53 =
[   0.84624772243093,   0.84624772243094]
>>
>>
>> for k = 3:10
  xx51p(k)=1.1+0.02*(k-3);
end

for k = 11:16
  xx51p(k)=1.25+0.01*(k-11);
end

xx51p(17)=1.305;

for k = 2:16
  Theta5p(xx51p(k),xx51p(k+1))
end

Theta5p(xx51p(17),xtilde52)

for k = 2:16
  Theta5p(xx51p(k+1),xx51p(k))
end

Theta5p(xtilde52,xx51p(17))

for k = 2:16
  h5p1(xx51p(k),xx51p(k+1))
end

h5p1(xx51p(17),xtilde52)

for k = 2:16
  vv5p1(xx51p(k),xx51p(k+1))
end

vv5p1(xx51p(17),xtilde52)
intval ans =
[   2.53005557432596,   2.53005557432602]
intval ans =
[   2.50819542152692,   2.50819542152698]
intval ans =
[   2.48177731670671,   2.48177731670678]
intval ans =
[   2.44995467971834,   2.44995467971840]
intval ans =
[   2.41156514979762,   2.41156514979769]
```

```
intval ans =
[   2.36495736954977,   2.36495736954984]
intval ans =
[   2.30767803903726,   2.30767803903733]
intval ans =
[   2.23585666369062,   2.23585666369070]
intval ans =
[   2.31387731378889,   2.31387731378897]
intval ans =
[   2.27012939129945,   2.27012939129952]
intval ans =
[   2.21927339353989,   2.21927339353996]
intval ans =
[   2.15900320128972,   2.15900320128980]
intval ans =
[   2.08558917756011,   2.08558917756019]
intval ans =
[   1.99235067619369,   1.99235067619378]
intval ans =
[   2.03265590384001,   2.03265590384013]
intval ans =
[   1.92199995992316,   1.92199995992334]
intval ans =
[   2.87922191503446,   2.87922191503452]
intval ans =
[   2.87106298779525,   2.87106298779531]
intval ans =
[   2.86062173390053,   2.86062173390060]
intval ans =
[   2.84767620243920,   2.84767620243926]
intval ans =
[   2.83195952762017,   2.83195952762025]
intval ans =
[   2.81315785207094,   2.81315785207100]
intval ans =
[   2.79092214258192,   2.79092214258199]
intval ans =
[   2.76492635739726,   2.76492635739733]
intval ans =
[   2.60061275051597,   2.60061275051604]
intval ans =
[   2.57562965057893,   2.57562965057900]
intval ans =
[   2.54790237356734,   2.54790237356742]
intval ans =
[   2.51705886983236,   2.51705886983244]
intval ans =
[   2.48275779291060,   2.48275779291068]
intval ans =
[   2.44495921496873,   2.44495921496882]
intval ans =
[   2.28855735681258,   2.28855735681268]
intval ans =
[   2.28922396728768,   2.28922396728785]
intval ans =
  1.0e+002 *
[  -1.01565618171488,  -1.01565618171479]
intval ans =
[ -80.63574223741732, -80.63574223741058]
```

```
intval ans =
[ -63.95656532865097, -63.95656532864533]
intval ans =
[ -50.54186468189202, -50.54186468188733]
intval ans =
[ -39.65819046884283, -39.65819046883890]
intval ans =
[ -30.75167852209927, -30.75167852209605]
intval ans =
[ -23.39662076868060, -23.39662076867788]
intval ans =
[ -17.25742314212705, -17.25742314212476]
intval ans =
[ -14.35465186640676, -14.35465186640497]
intval ans =
[ -12.09352581872649, -12.09352581872487]
intval ans =
[ -10.00020329053763, -10.00020329053611]
intval ans =
[  -8.04869939832424,  -8.04869939832285]
intval ans =
[  -6.20944095926728,  -6.20944095926600]
intval ans =
[  -4.44245204836912,  -4.44245204836792]
intval ans =
[  -3.93560192450687,  -3.93560192450578]
intval ans =
[  -2.80289182457329,  -2.80289182457162]
intval ans =
[ -36.28656439118602, -36.28656439116919]
intval ans =
[ -27.36212215923211, -27.36212215921845]
intval ans =
[ -20.27817931141765, -20.27817931140622]
intval ans =
[ -14.62816879402557, -14.62816879401612]
intval ans =
[ -10.10511324559011, -10.10511324558226]
intval ans =
[  -6.47397531851405,  -6.47397531850763]
intval ans =
[  -3.55172509041479,  -3.55172509040937]
intval ans =
[  -1.19198657063779,  -1.19198657063326]
intval ans =
[  -4.67032631365826,  -4.67032631365470]
intval ans =
[  -3.65583321197128,  -3.65583321196808]
intval ans =
[  -2.74360983780925,  -2.74360983780626]
intval ans =
[  -1.92068900135242,  -1.92068900134969]
intval ans =
[  -1.17195773506850,  -1.17195773506603]
intval ans =
[  -0.47565056529120,  -0.47565056528888]
intval ans =
[  -1.96878381461315,  -1.96878381461107]
intval ans =
```

```
[  -1.09263094285403,  -1.09263094285099]
>>
>>
>> xx51p(19)=1.317;
xx51p(20)=1.323;

Theta5p(xtilde52,xx51p(19))
Theta5p(xx51p(19),xx51p(20))
Theta5p(xx51p(20),xtilde51)

Theta5p(xx51p(19),xtilde52)
Theta5p(xx51p(20),xx51p(19))
Theta5p(xtilde51,xx51p(20))

h5p2(xtilde52,xx51p(19))
h5p2(xx51p(19),xx51p(20))
h5p2(xx51p(20),xtilde51)

vv5p2(xtilde52,xx51p(19))
vv5p2(xx51p(19),xx51p(20))
vv5p2(xx51p(20),xtilde51)
intval ans =
[   1.84280874445733,   1.84280874445752]
intval ans =
[   1.68332336741943,   1.68332336741960]
intval ans =
[   1.63318517295109,   1.63318517295148]
intval ans =
[   2.22785140000702,   2.22785140000721]
intval ans =
[   2.20446118157256,   2.20446118157272]
intval ans =
[   2.05168727695358,   2.05168727695389]
intval ans =
[  -1.91225591303950,  -1.91225591303793]
intval ans =
[  -0.70830240359305,  -0.70830240359196]
intval ans =
[  -0.32097714442255,  -0.32097714442056]
intval ans =
[  -0.99616831623402,  -0.99616831623091]
intval ans =
[  -0.38008113413083,  -0.38008113412892]
intval ans =
[  -1.12256685919691,  -1.12256685919373]
>>
>>
>> xx51p(22)=1.329;

Theta5p(xtilde51,xx51p(22))
Theta5p(xx51p(22),xtilde51)
h5p3(xtilde51,xx51p(22))
vv5p3(xtilde51,xx51p(22))

Theta5p1(xx51p(22),0.78+epsilon5)
Theta5p2(0.78+epsilon5,xx51p(22))

h5p33(xx51p(22),0.78+epsilon5)
vv5p33(xx51p(22),0.78+epsilon5)
```

```
intval ans =
[   1.50934837796702,   1.50934837796748]
intval ans =
[   1.96982677327354,   1.96982677327401]
intval ans =
[   0.27504884983239,   0.27504884983452]
intval ans =
[  -1.19415582074728,  -1.19415582074336]
intval ans =
[   1.27361771560215,   1.27361771560234]
intval ans =
[   1.92281761797388,   1.92281761797415]
intval ans =
[   1.15837200377452,   1.15837200377547]
intval ans =
[  -0.99885578205305,  -0.99885578205104]
>>
>>
>> Q(x51m)
0.25*0.7/DQ(x51m)

x511m=1.293;
x511m-x51m
y5m(x511m)

infsup(1,1)*0.0175/sqrt(2)

s54=imag(omega)/real(x51m-omega);
(1.2-real(a5))^2+(s54*(1.2-x51m)-imag(a5))^2-epsilon5^2
intval ans =
[   0.95142777570186,   0.95142777570197]
intval ans =
[   0.01761655418104,   0.01761655418105]
intval ans =
[   0.00436835103378,   0.00436835103379]
intval ans =
[   0.01090918069862,   0.01090918069863]
intval ans =
[   0.01237436867076,   0.01237436867077]
intval ans =
[  -0.02649212774476,  -0.02649212774474]
>>
>>
>> xx51m=ones(1,11);
xx51m(1)=1.293;
xx51m(2)=1.297;
xx51m(3)=1.3;
xx51m(4)=1.305;
xx51m(5)=1.31;
xx51m(6)=1.315;
xx51m(7)=1.32;
xx51m(8)=1.325;
xx51m(9)=1.327;
xx51m(10)=1.329;

for k = 1:9
  Theta5m(xx51m(k+1),xx51m(k))
end
```

```
Theta5m1110=theta51m(xx51m(10))+theta52m(xx51m(10))-atan(imag(a5)/(0.78+epsilon5))-4
*atan(imag(a5-omega)/(0.78+epsilon5-real(omega)))-4*atan(imag(a5+omega)/(0.78
+epsilon5-real(omega)))

for k = 1:9
  Theta5m(xx51m(k),xx51m(k+1))
end

6*atan(imag(a5)/(0.78+epsilon5+1))+4*atan(imag(a5)/(0.78+epsilon5-1))-theta53m(xx51m
(10))-theta54m(xx51m(10))-theta55m(xx51m(10))
intval ans =
[   0.04714960557642,   0.04714960557654]
intval ans =
[   0.16444647409129,   0.16444647409142]
intval ans =
[   0.20331290844261,   0.20331290844274]
intval ans =
[   0.32921388879029,   0.32921388879043]
intval ans =
[   0.45721826990355,   0.45721826990370]
intval ans =
[   0.58645002293139,   0.58645002293157]
intval ans =
[   0.71051527683936,   0.71051527683957]
intval ans =
[   0.99270020869474,   0.99270020869500]
intval ans =
[   1.05029675112201,   1.05029675112237]
intval Theta5m1110 =
[   1.08331220089579,   1.08331220089612]
intval ans =
[   0.27993111227869,   0.27993111227882]
intval ans =
[   0.34640296161999,   0.34640296162012]
intval ans =
[   0.52369903564912,   0.52369903564926]
intval ans =
[   0.67712548516602,   0.67712548516617]
intval ans =
[   0.84407889014940,   0.84407889014956]
intval ans =
[   1.03441418037980,   1.03441418037998]
intval ans =
[   1.27574007924909,   1.27574007924932]
intval ans =
[   1.28968885654327,   1.28968885654355]
intval ans =
[   1.45884131065314,   1.45884131065354]
intval ans =
[   1.74809534058688,   1.74809534058706]
>>
>>
>> for k = 1:9
  Xi5mtilde(xx51m(k),xx51m(k+1))
end

Xi5mtilde1011=((0.78+epsilon5+1)^2+(imag(a5))^2)^3*((0.78+epsilon5-1)^2+(imag(a5))^2)
^2/(xi53m(xx51m(10))*((0.78+epsilon5-real(omega))^2+(imag(a5-omega))^2)^2*xi55m(xx51m
(10)))
```

```
intval ans =
[   1.09784895543863,   1.09784895543870]
intval ans =
[   1.13044235825673,   1.13044235825680]
intval ans =
[   1.24966767965213,   1.24966767965222]
intval ans =
[   1.36591561066652,   1.36591561066662]
intval ans =
[   1.51813938607473,   1.51813938607486]
intval ans =
[   1.73144957602963,   1.73144957602980]
intval ans =
[   2.07596185037660,   2.07596185037687]
intval ans =
[   2.15635131640490,   2.15635131640526]
intval ans =
[   2.49081646174483,   2.49081646174545]
intval Xi5mtilde1011 =
[   3.21086664670092,   3.21086664670128]
>>
>>
>> for k = 1:9
  Xi5mtilde(xx51m(k),xx51m(k+1))*cos(Theta5m(xx51m(k+1),xx51m(k)))
end

Xi5mtilde1011*cos(Theta5m1110)
intval ans =
[   1.09662887555170,   1.09662887555177]
intval ans =
[   1.11519169848235,   1.11519169848244]
intval ans =
[   1.22392830876258,   1.22392830876270]
intval ans =
[   1.29256155101771,   1.29256155101787]
intval ans =
[   1.36220213976573,   1.36220213976594]
intval ans =
[   1.44214254583131,   1.44214254583162]
intval ans =
[   1.57363285100113,   1.57363285100161]
intval ans =
[   1.17829595209579,   1.17829595209646]
intval ans =
[   1.23871694430036,   1.23871694430142]
intval ans =
[   1.50398458748457,   1.50398458748562]
>>
>>
>> % The INTLAB code to verify the numerical estimates in the paper:
% Parabolic and near-parabolic renormalizations for local degree three
% arXiv: 1510.00043v5, 2024.
>>
```