ON TOURNAMENTS OF SMALL ORDERS AND THEIR APPLICATIONS

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Abstract. In this paper, we generate all nonisomorphic tournaments of order at most nine, all nonisomorphic almost regular tournaments of order 10 and all nonisomorphic regular tournaments of order 11. For each of these tournaments, we have given its score-list, connectivity, diameter, the minimal number of feedbacks, automorphisms and spectra. Moreover, we have verified the well-known Kelly's Conjecture for $n = 2k + 1 \leq 11$. And we also determine the n-universal tournaments for $n \leq 6$. However, several related results are given and some related open problems are raised.

Key words. Tournaments, applications.

1 Introduction

In [1], D. Cvetković and M. Petrić illustrated a table of 112 connected graphs on six vertices. Several data such as the spectrum, and its main part, coefficients of the characteristic polynomial and of the matching polynomial, number of cycles, etc., are given for each graph in the table. In [2], D. Cvetković and Z. Radosavljević gave a table of exactly 250 regular graphs on at most ten vertices. As for tournaments, in the appendix of [3], the author illustrated all nonisomorphic tournaments T_n ($n \leq 6$), their score-lists, the number of ways labeling their nodes, and their automorphism groups. In this paper, we are concerned with tournaments of order $n \leq 9$. Tournaments form a large class of directed graphs, they provide a rich source for combinatorial investigations and for various models in applied situations.

Using a program "NAUTY" written by B. D. McKay[4], we generate all nonisomorphic tournaments T_n ($n \leq 9$) with the aid of a computer. It seems that when $n \geq 10$, illustrating all nonisomorphic tournaments T_n will be out of calculation. However, we have illustrated all 13333 nonisomorphic almost regular tournaments of order 10 and all 1223 nonisomorphic regular tournaments of order 11. For each of these tournaments, we have given its score-lists, connectivity, diameter, the minimal number of feedbacks, automorphisms and spectra. Moreover, several conclusions are drawn, implied by the data of these nonisomorphic tournaments.

Let $T(n)$ denote the number of nonisomorphic tournaments T_n. The values of $T(n)$ for $1 \leq n \leq 12$ are given in Table 1[3];

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Table 1 $T(n)$, the number of nonisomorphic tournaments T_n

n	1	2	3	4	5	6
$T(n)$	1	1	2	4	12	56
n	7	8	9	10	11	12
$T(n)$	456	6,880	191,536	9,733,056	903,753,248	154,108,311,168

2 Terminology and Notation

A tournament is a directed graph in which every pair of vertices is joined by exactly one arc. If the arc joining vertices v and w is directed from v to w, then v is said to dominate w (symbolically, $v \to w$). The set of vertices dominated by v is denoted by $N^+(v)$, and the set of vertices which dominate v is denoted by $N^-(v)$. We define the outdegree $d^+(v) = |N^+(v)|$ and the indegree $d^-(v) = |N^-(v)|$. The score $s(v)$ of the vertex v is the number of vertices dominated by v. A tournament is regular if all vertices have equal scores and a tournament is almost regular if the difference of scores between any two vertices is at most 1 and there exist at least two vertices such that the difference of scores between them is 1. The score-list of a tournament is the list of the scores of the vertices, usually arranged in non-decreasing order. A tournament is strong if from each vertex there are directed paths to all other vertices. Two tournaments T_1 and T_2 are said to be isomorphic if there is a bijection f from $V(T_1)$ to $V(T_2)$ such that for every arc uv of T_1, $f(u)f(v)$ is an arc of T_2.

A tournament of order n is called an n-tournament, and the induced subtournament of T with S as its set of vertices will be denoted by $T[S]$. The converse T' of a tournament T has the same vertex-set as T, but every arc is conversed (that is, if vw is an arc in T, then wv is an arc of T'). The adjacency matrix of a tournament T_n is the $n \times n$ matrix $M(T_n) = (a_{ij})$ in which a_{ij} is 1 if $p_i \to p_j$ and 0 otherwise, where $V(T_n) = \{p_1, p_2, \cdots, p_n\}$.

A simple path (cycle, resp.) in a digraph is antidirected if every two adjacent arcs of the path (cycle, resp.) have opposite orientations. An antidirected Hamiltonian path (cycle, resp.) (abbreviated to ADH-path (ADH-cycle, resp.)), is a simple antidirected path (cycle, resp.) which contains all vertices.

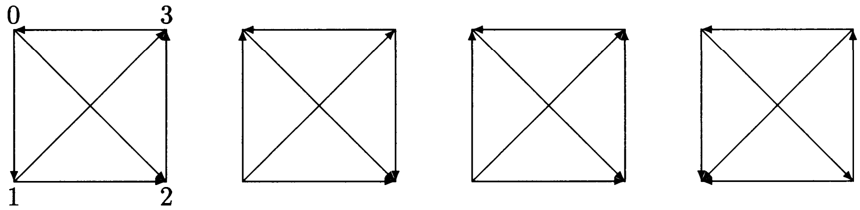
A transitive tournament is a tournament which contains no directed cycle. TT_n denotes a transitive tournament with n vertices. T_3^c and T_5^c are the unique regular tournaments with three and five vertices, respectively. T_7^c is a regular tournament with seven vertices $v_0, v_1, \cdots, v_6$, and $v_i \to v_j$ if and only if $i - j \pmod 7 \in \{1, 2, 4\}$. $T_4^c = T_5^c - x$ and $T_6^c = T_7^c - x$. A vertex v is called a starting (terminating, resp.) vertex in T_n if there exists an ADH-path $v \to v_1 \leftarrow \cdots$ ($v \leftarrow v_1 \to \cdots$, resp.). If v is both a starting and a terminating vertex, then v is called a double point.

In this paper, an n-tournament is represented by an array of n sets, the i-th set gives the vertices which are dominated by vertex i. For example, the tournament in Figure 1(a) will be represented by

0 : 1 2;
1 : 2 3;
2 : 3;
3 : 0.

3 A Table of Tournaments of Small Orders

By calling a program "NAUTY" written by B. D. McKay[4], we generate all nonisomorphic tournaments T_n ($n \leq 9$) with the aid of a computer. It seems that when $n \geq 10$, illustrating all nonisomorphic tournaments T_n will be out of calculation. However, we have illustrated all 13333 nonisomorphic almost regular tournaments of order 10 and all 1223 nonisomorphic regular tournaments of order 11. For each of these tournaments, we have given its score-lists, connectivity, diameter, the minimal number of feedbacks, automorphisms and spectra (for details, the reader can contact the first author).



(a) (b) (c) (d)

Figure 1 Nonisomorphic tournaments of order 4

4 Regular Tournaments and Kelly's Conjecture

4.1 Regular tournaments

Since regular tournaments seem to have good characterization, a lot of authors are interested in regular tournaments. In the following, we will illustrate all nonisomorphic regular tournaments with at most nine vertices. Those nonisomorphic regular tournaments are saved in a CD.

0: 1;	0: 1 2;
1: 2;	1: 2 3;
2: 0.	2: 3 4;
	3: 4 0;
	4: 0 1.

Figure 2 Regular tournaments of order three and five

0: 1 5 6;	0: 3 5 6;	0: 3 5 6;
1: 4 5 6;	1: 0 2 6;	1: 0 4 5;
2: 0 1 5;	2: 0 3 5;	2: 0 1 6;
3: 0 1 2;	3: 1 4 5;	3: 1 2 5;
4: 0 2 3;	4: 0 1 2;	4: 0 2 3;
5: 3 4 6;	5: 1 4 6;	5: 2 4 6;
6: 2 3 4.	6: 2 3 4.	6: 1 3 4.
(a)	(b)	(c)

Figure 3 Regular tournaments of order seven.

4.2 On Kelly's conjecture

A conjecture attributed to P. Kelly (see [3], p.7) is as follows:

Conjecture 4.1 The arcs of every regular tournament can be partitioned into arc-disjoint Hamiltonian cycles.

This conjecture was verified for $n \leq 9$ by Alspach. Morever, R. Häggkvist claims that he has proved that Kelly's Conjecture is true for a sufficiently large n ([5]), but unfortunately his proof has never been written up in full details.

0: 4 5 6 7;	0: 4 5 7 8;	0: 1 5 7 8;	0: 1 5 6 8;	0: 1 5 6 7;
1: 0 4 5 6;	1: 0 4 5 6;	1: 3 5 6 7;	1: 3 5 6 7;	1: 3 5 6 8;
2: 0 1 4 5;	2: 0 1 4 5;	2: 0 1 5 6;	2: 0 1 5 7;	2: 0 1 5 7;
3: 0 1 2 4;	3: 0 1 2 4;	3: 0 2 5 6;	3: 0 2 5 6;	3: 0 2 5 6;
4: 5 6 7 8;	4: 5 6 7 8;	4: 0 1 2 3;	4: 0 1 2 3;	4: 0 1 2 3;
5: 3 6 7 8;	5: 3 6 7 8;	5: 4 6 7 8;	5: 4 6 7 8;	5: 4 6 7 8;
6: 2 3 7 8;	6: 0 2 3 7;	6: 0 4 7 8;	6: 2 4 7 8;	6: 2 4 7 8;
7: 1 2 3 8;	7: 1 2 3 8;	7: 2 3 4 8;	7: 0 3 4 8;	7: 1 3 4 8;
8: 0 1 2 3.	8: 1 2 3 6.	8: 1 2 3 4.	8: 1 2 3 4.	8: 0 2 3 4.
(1)	(2)	(3)	(4)	(5)
0: 1 5 7 8;	0: 1 5 7 8;	0: 1 5 7 8;	0: 1 6 7 8;	0: 1 6 7 8;
1: 3 5 6 8;	1: 3 5 6 7;	1: 3 5 6 8;	1: 3 5 6 7;	1: 3 5 6 7;
2: 0 1 5 7;	2: 0 1 5 8;	2: 0 1 5 6;	2: 0 1 5 8;	2: 0 1 5 6;
3: 0 2 5 6;	3: 0 2 5 6;	3: 0 2 5 7;	3: 0 2 5 6;	3: 0 2 5 8;
4: 0 1 2 3;	4: 0 1 2 3;	4: 0 1 2 3;	4: 0 1 2 3;	4: 0 1 2 3;
5: 4 6 7 8;	5: 4 6 7 8;	5: 4 6 7 8;	5: 0 4 6 7;	5: 0 4 6 7;
6: 0 2 4 7;	6: 0 2 4 7;	6: 0 3 4 7;	6: 2 4 7 8;	6: 3 4 7 8;
7: 1 3 4 8;	7: 2 3 4 8;	7: 1 2 4 8;	7: 2 3 4 8;	7: 2 3 4 8;
8: 2 3 4 6.	8: 1 3 4 6.	8: 2 3 4 6.	8: 1 3 4 5.	8: 1 2 4 5.
(6)	(7)	(8)	(9)	(10)
0: 1 6 7 8;	0: 1 5 6 8;	0: 1 5 6 7;	0: 3 5 7 8;	0: 4 5 6 7;
1: 3 5 7 8;	1: 3 6 7 8;	1: 3 5 6 7;	1: 0 2 6 7;	1: 0 4 6 8;
2: 0 1 5 8;	2: 0 1 5 7;	2: 0 1 7 8;	2: 0 3 5 8;	2: 0 1 4 5;
3: 0 2 5 6;	3: 0 2 5 6;	3: 0 2 5 6;	3: 1 4 5 6;	3: 0 1 2 4;
4: 0 1 2 3;	4: 0 1 2 3;	4: 0 1 2 3;	4: 0 1 2 7;	4: 5 6 7 8;
5: 0 4 6 7;	5: 1 4 6 7;	5: 2 4 6 8;	5: 1 4 6 8;	5: 1 3 6 7;
6: 1 2 4 7;	6: 2 4 7 8;	6: 2 4 7 8;	6: 0 2 4 7;	6: 2 3 7 8;
7: 2 3 4 8;	7: 0 3 4 8;	7: 3 4 5 8;	7: 2 3 5 8;	7: 1 2 3 8;
8: 3 4 5 6.	8: 2 3 4 5.	8: 0 1 3 4.	8: 1 3 4 6.	8: 0 2 3 5.
(11)	(12)	(13)	(14)	(15)

Figure 4 Regular tournaments of order nine.

More than thirty years have passed, Conjecture 4.1 has not been proved. It seems that Conjecture 4.1 is very difficult to prove, we confirm this conjecture for tournament of order 11 or less.

The numbers of distinct Hamiltonian cycles in Figure 3 are 17, 15 and 24. Let $N(i)$ be the distinct Hamiltonian cycles in Figure 4(i). Then

Table 2 The number of distinct Hamiltonian cycles in regular tournaments of order 9

$N(1)$	$N(2)$	$N(3)$	$N(4)$	$N(5)$
222	224	221	230	224
$N(6)$	$N(7)$	$N(8)$	$N(9)$	$N(10)$
222	224	225	231	231
$N(11)$	$N(12)$	$N(13)$	$N(14)$	$N(15)$
243	249	225	207	228

Let $HN(k)$ be the minimum number of distinct Hamiltonian cycles in a regular tournament of order $n = 2k + 1$ and let $R(k) = HN(k)/k$. Thus we have $HN(3) = 5$, $HN(4) = 207$, $HN(5) = 4899$, $R(3) = 5$, $R(4) = 51.75 > R(3) \times 10$, $R(5) = 979.8 > R(4) \times 10$. From the order of an increase in $R(k)$, we believe strongly that the Kelly's Conjecture is true once more.

5 Some Applications

5.1 Spectra and spectral radius of tournaments

Let A be a square matrix. Then the determinant of A is denoted by $\det(A)$. Let T_n be an n-tournament, and let M be an adjacency matrix of T_n. The characteristic polynomial of T_n is $\det(\lambda I - M)$, and the eigenvalues of M are also called the eigenvalues of T_n. Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the eigenvalues of T_n, where $|\lambda_1| \geq |\lambda_2| \geq \cdots \geq |\lambda_n|$. Then $|\lambda_1|$ is called the spectral radius of T_n, denoted by $\rho(T_n)$. The spectrum is denoted by $Sp(T_n) = [\lambda_1, \lambda_2, \cdots, \lambda_n]$. We define $\bar{\rho}_n = \max\{\rho(T_n)\}$, with T_n being over all nonisomorphic tournaments of order n. And define $\tilde{\rho}_n = \min\{\rho(T_n)\}$.

It can be proved[6] that if $n =$ odd, then $\bar{\rho}_n = (n-1)/2$. For $n =$ even, R. A. Brualdi and Li Qiao[6] raised the following

Conjecture 5.1 $\rho(T_n) = \bar{\rho}_n$ if and only if T_n is almost regular and $\bar{\rho}_n = \rho(\overline{T}_n)$, where

$$A(\overline{T}_n) = \begin{bmatrix} M & M^{\mathrm{T}} \\ M^{\mathrm{T}} + I & M \end{bmatrix}$$

and

$$M = \begin{bmatrix} 0 & 1 & \cdots & 1 & 1 \\ 0 & 0 & 1 & \cdots & 1 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}.$$

Here we give some support to this Conjecture. $\bar{\rho}_4 = 1.3534$, it can be reached only if

$$\overline{T}_4 = \begin{array}{ll} 0: & 1\ 2; \\ 1: & 2\ 3; \\ 2: & 3; \\ 3: & 0. \end{array}$$

$\bar{\rho}_6 = 1.83929$, it can be reached only if

$$\overline{T}_6 = \begin{array}{ll} 0: & 1\ 5; \\ 1: & 4\ 5; \\ 2: & 0\ 1\ 5; \\ 3: & 0\ 1\ 2; \\ 4: & 0\ 2\ 3; \\ 5: & 3\ 4. \end{array}$$

$\bar{\rho}_8 = 3.45135$, it can be reached only if

$$\overline{T}_8 = \begin{array}{ll} 0: & 4\ 5\ 6\ 7; \\ 1: & 0\ 4\ 5\ 6; \\ 2: & 0\ 1\ 4\ 5; \\ 3: & 0\ 1\ 2\ 4; \\ 4: & 5\ 6\ 7; \\ 5: & 3\ 6\ 7; \\ 6: & 2\ 3\ 7; \\ 7: & 1\ 2\ 3. \end{array}$$

These $A(\overline{T}_i)$ $i \in \{4, 6, 8\}$ are exactly of the form (up to an isomorphism) stated in the above Conjecture.

They also raised the following

Conjecture 5.2 If T_n^1 and T_n^2 are two non-isomorphic almost regular tournaments, then $\rho(T_n^1) \neq \rho(T_n^2)$.

We disprove this Conjecture. The following two nonisomorphic tournaments T_8^1 and T_8^2 are almost regular, but $\rho(T_n^1) = \rho(T_n^2)$.

T_8^1	T_8^2
0 : 4 5 7;	0 : 4 5 7;
1 : 0 4 5 6;	1 : 0 4 5 6;
2 : 0 1 4 5;	2 : 0 1 4 5;
3 : 0 1 2 4;	3 : 0 1 2 4;
4 : 5 6 7;	4 : 5 6 7;
5 : 3 6 7;	5 : 3 6 7;
6 : 0 2 3 7;	6 : 0 2 3;
7 : 1 2 3.	7 : 1 2 3 6.

In fact, both of the above tournaments have the same characteristic polynomials:

$$x^8 - 20x^5 - 46x^4 - 70x^3 - 60x^2 - 29x - 6,$$

hence they have the same spectral radius.

Conjecture 5.3 $\tilde{\rho}_n = \rho(\widetilde{T}_n)$, where

$$A(\widetilde{T}_n) = \begin{bmatrix} 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & \ddots & \cdot & \cdot & 0 \\ 1 & 0 & 0 & 1 & \ddots & \cdot & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 1 & \cdots & 1 & \ddots & \ddots & \ddots & 0 \\ 1 & 1 & \cdots & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 & 0 \end{bmatrix}.$$

Here we give some support to this Conjecture:

Table 3 $\tilde{\rho}_n$

$\tilde{\rho}_3$	$\tilde{\rho}_4$	$\tilde{\rho}_5$	$\tilde{\rho}_6$	$\tilde{\rho}_7$	$\tilde{\rho}_8$	$\tilde{\rho}_9$
1.000	1.39534	1.65930	1.83929	1.96702	2.06064	2.83585

When $\tilde{\rho}_n = \rho(T_n)$ $n = 3, 4, \cdots, 9$, $A(T_n)$ is exactly of the form (up to an isomorphism) stated in the above Conjecture.

5.2 Universal tournaments

A tournament T_N is said to be n-universal ($n \leq N$) if every tournament T_n is isomorphic to some subtournament of T_N. For every positive integer n, let $\lambda(n)$ denote the least integer N for which there exists an n-universal tournament T_N. It is clear that $\lambda(n)$ is finite, since any tournament that contains disjoint copies of all the different tournaments T_n is n-universal.

Moon[3] obtained the bound for $\lambda(n)$.

Theorem 5.1

$$2^{(1/2)(n-1)} \leq \lambda(n) \leq \begin{cases} n2^{(1/2)(n-1)} & \textit{if } n \textit{ is odd}, \\ \dfrac{3}{2\sqrt{2}} n2^{(1/2)(n-1)} & \textit{if } n \textit{ is even}. \end{cases}$$

However, we have determined the exact values of $\lambda(n)$ for $n \leq 6$.

Theorem 5.2 *The exact values of* $\lambda(n)$ *for* $n \leq 6$ *are*:

n	1	2	3	4	5	6
$\lambda(n)$	1	2	4	5	8	10

In 1971, B. Grünbaum introduced the ADH-paths and ADH-cycles in[7] and proved that every tournament, except T_3^c, T_5^c and T_7^c.has an ADH-path. A simpler proof with some additional results was given by M. Rosenfeld in [8].

Theorem 5.3[7] *If a tournament* T_n *with odd order has an ADH-path, then* T_n *has a double point.*

Theorem 5.4[8] *Every tournament, except* T_3^c, T_5^c *and* T_7^c, *has an ADH-path.*

Theorem 5.5[9] (a) *If* $n = 2k$, *then* TT_n *has an ADH-path starting at* i $(i \neq n)$ *and terminating at* j *except for the following cases*:

(i) $j = 1$,

(ii) $i = 1, j = 2$ $(n > 2)$,

(iii) $i = 2k-1, j = 2k$.

(b) *If* $n = 2k+1$, *then* TT_n *has an ADH-path with* i *and* j *as the starting vertices if* $i, j \neq 2k+1$ *and* $\{i, j\} \neq \{2k-1, 2k\} (n > 3)$.

M. R. Rosenfeld[8] proved that any tournament of odd order $n = 2k+1 \geq 9$ contains a double point. With the aid of a computer, we get

Theorem 5.6 *Let* T_n *be a tournament of order* $n \in \{4, 5, 6, 7, 8\}$. *Then each* T_n *contains a double point except* $T_n = T_4^c, T_5^c, T_6^c, T_7^c$.

By Theorems 5.3 and 5.4, we know that if $n \geq 9$ and n is odd, then every tournament T_n has a double point. Also, by an exhaustive search on all the nonisomorphic tournaments of order 8 with a computer, we find each T_8 has a double point. So we raise the following conjecture:

Conjecture 5.4 If $n \geq 8$, then each tournament of order n contains a double point.

Since a tournament T_n can have an ADH-cycle only if n is even, and T_8 which contains a T_7^c can't have any ADH cycle, B. Grünbaum conjectured that every tournament T_n with $n = 2k \geq 10$ has an ADH-cycle. The conjecture was proved by C. Thomassen[10] for $n \geq 50$ and by M. Rosenfeld[9] for $n \geq 26$ and by V. Petrovic[11] for $n \geq 16$.As a major application of the table of tournaments of order at most nine, this conjecture is completely solved. That is, we have

Theorem 5.7[12] *Every* T_n $(n = 2k \geq 10)$ *contains an ADH-cycle.*

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