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The Ramsey numbers of trees versus W_6 or W_7 ☆

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Abstract

Let T_n denote a tree of order n and W_m a wheel of order $m+1$. In this paper, we show the Ramsey numbers $R(T_n, W_6) = 2n - 1 + \mu$ for $n \geq 5$, where $\mu = 2$ if $T_n = S_n$, $\mu = 1$ if $T_n = S_n(1, 1)$ or $T_n = S_n(1, 2)$ and $n \equiv 0 \pmod 3$, and $\mu = 0$ otherwise; $R(T_n, W_7) = 3n - 2$ for $n \geq 6$.

Keywords: Ramsey number; Tree; Wheel

1. Introduction

All graphs considered in this paper are finite simple graphs without loops. For two given graphs G_1 and G_2, the *Ramsey number* $R(G_1, G_2)$ is the smallest positive integer n such that for any graph G of order n, either G contains G_1 or $\overline{G}$ contains G_2, where $\overline{G}$ is the complement of G. Let G be a graph and m be a positive integer. We use mG to denote m vertex disjoint copies of G. A *path* and a *cycle* of order n are denoted by P_n and C_n, respectively. A *star* S_n $(n \geq 3)$ is a bipartite graph $K_{1,n-1}$. A *complete graph* of order n is denoted by K_n. A *wheel* $W_n = K_1 + C_n$ is a graph of $n + 1$ vertices, where K_1 is called the *hub* of the wheel. $S_n(l, m)$ is a tree of order n obtained from $S_{n-l\times m}$ by subdividing each of l chosen edges m times. $S_n(l)$ is a tree of order n obtained from an S_l and an S_{n-l} by adding an edge joining the centers of them. $S_n[l]$ is a tree of order n obtained from an

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S_l and an S_{n-l} by adding an edge joining a vertex of degree one of S_l to the center of S_{n-l}. Define

$$\mathcal{T} = \{S_n \mid n \geq 5\} \cup \{S_n(1,1) \mid n \geq 5\} \cup \{S_n(1,2) \mid n \geq 6 \text{ and } n \equiv 0 \pmod 3\}.$$

For a tree T, we define $L(T) = \{v \mid v \in V(T) \text{ and } d(v) = 1\}$. Let $V \subseteq L(T)$ and $|V| = k$. Write $T_V = T - V$. If $T_V \notin \mathcal{T}$, we call V a k-*deletable set*. If $k = 2$ and $|N(V)| = 2$, we call V a II-*set*. If $k = 3$ and $|N(V)| = 3$, we call V a III-*set*. If $k = 3$ and $|N(V)| = 2$, we call V a IV-*set*. If V is a II-*set* and $T_V \notin \mathcal{T}$, we call V a II-*deletable set*. Similarly, we can define III-*deletable* and IV-*deletable sets*. Terminology and notations not defined here can be found in [2].

In [1], Baskoro et al. obtain the following.

Theorem 1 ([1]). *Let T_n be a tree of order n other than S_n. Then $R(T_n, W_4) = 2n - 1$ for $n \geq 3$; $R(T_n, W_5) = 3n - 2$ for $n \geq 4$.*

Motivated by Theorem 1, Baskoro et al. [1] pose the following.

Conjecture 1. *Let T_n be a tree of order n other than S_n and $n \geq m-1$. Then $R(T_n, W_m) = 2n - 1$ for even $m \geq 6$; $R(T_n, W_m) = 3n - 2$ for odd $m \geq 7$.*

In [3], we show Conjecture 1 holds for $T_n = P_n$.

Theorem 2 ([3]). *$R(P_n, W_m) = 3n - 2$ for m odd and $n \geq m - 1 \geq 2$; $R(P_n, W_m) = 2n - 1$ for m even and $n \geq m - 1 \geq 3$.*

In [4], we obtain the following.

Theorem 3 ([4]). *$R(S_n, W_6) = 2n + 1$ for $n \geq 3$; $R(S_n, W_m) = 3n - 2$ for m odd and $n \geq m - 1 \geq 2$.*

Using Theorem 3, we consider $R(T_n, W_6)$ for $\Delta(T_n) \geq n - 3$ in [5] and the following are established.

Theorem 4 ([5]). *$R(S_n(1,1), W_6) = 2n$ for $n \geq 4$.*

Theorem 5 ([5]). *$R(S_n(1,2), W_6) = 2n$ for $n \geq 6$ and $n \equiv 0 \pmod 3$.*

Theorem 6 ([5]). *$R(S_n(3), W_6) = R(S_n(2,1), W_6) = 2n - 1$ for $n \geq 6$; $R(S_n(1,2), W_6) = 2n - 1$ for $n \geq 6$ and $n \not\equiv 0 \pmod 3$.*

By Theorems 4 and 5, we can see that Conjecture 1 is not true when $m = 6$. In fact, as pointed out in [5], for even m, $R(T_n, W_m)$ is a function related to both n and m. However, we believe that $R(T_n, W_6) = 2n - 1$ for $T_n \notin \mathcal{T}$.

In [6], we evaluate $R(T_n, W_6)$ for $5 \leq n \leq 8$ and get the following.

Theorem 7 ([6]). *Let $T_n \notin \mathcal{T}$ be a tree of order n and $5 \leq n \leq 8$, then $R(T_n, W_6) = 2n - 1$.*

In [7], we consider $R(T_n, W_6)$ for T_n without certain deletable sets and establish the following.

Theorem 8 ([7]). *Let $T \notin \mathcal{T}$ be a tree of order $n \geq 9$. If T contains no II-deletable set, or $|L(T)| \geq 3$ and T contains neither III-deletable set nor IV-deletable set, or $|L(T)| \geq 4$ and T contains no IV-deletable set, then $R(T, W_6) = 2n - 1$.*

In this paper, we will determine $R(T_n, W_6)$ for all $T_n \notin \mathcal{T}$ and $n \geq 5$. On the other hand, we will consider the conjecture in the case where m is odd. As a special case, this paper will determine $R(T_n, W_7)$.

Let T_n be a tree of order n. The main results of this paper are the following.

Theorem 9. *$R(T_n, W_6) = 2n - 1 + \mu$ for $n \geq 5$, where $\mu = 2$ if $T_n = S_n$, $\mu = 1$ if $T_n = S_n(1, 1)$ or $T_n = S_n(1, 2)$ and $n \equiv 0 \pmod 3$, and $\mu = 0$ otherwise.*

Theorem 10. *$R(T_n, W_7) = 3n - 2$ for $n \geq 6$.*

By Theorem 10, we can see that Conjecture 1 holds for $m = 7$. For odd $m \geq 9$, the conjecture is still alive. Although the conjecture is not true for even m in general, we believe it holds for odd m.

2. Some lemmas

In order to prove the main results of this paper, we need the following lemmas.

Lemma 1 ([5]). *Let G be a graph of order $2n - 1 \geq 7$ and (U, V) a partition of $V(G)$ with $|U| \geq 3$ and $|V| \geq 4$. Suppose $u_i \in U$ and $N_V(u_i) = \emptyset$, $1 \leq i \leq 3$. If $\overline{G}$ contains no W_6, then $\delta(G[V]) \geq |V| - 3$.*

Lemma 2 ([7]). *Let G be a graph of order $2n - 1$. If $\alpha(G) \leq 2$, then G contains all trees of order n.*

Ore showed in [8] that if a graph on n vertices in which the degree sum of any two nonadjacent vertices is at least $n + 1$, then G is Hamilton-connected. From this result we can get easily the following.

Lemma 3. *Let G be a graph of order n. If $\delta(G) \geq n/2 + 1$, then G is Hamilton-connected.*

Lemma 4. *Let G be a graph of order $n \geq 9$. If $\alpha(G) \leq 2$ and $\delta(G) \geq n - 3$, then G contains all trees T of order n with $|L(T)| = 3$.*

Proof. If $\alpha(G) = 1$, then it holds trivially. Hence we may assume $\alpha(G) = 2$. Let T be a given tree of order n with $|L(T)| = 3$. Obviously, $\Delta(T) = 3$ and T has only one vertex of degree 3. Let $v \in V(G)$ and $G_0 = G - v$. Since $n \geq 9$ and $\delta(G) \geq n - 3$, we have $\delta(G_0) \geq (n - 3) - 1 \geq (n - 1)/2 + 1 = |G_0|/2 + 1$ which implies G_0 is Hamilton-connected by Lemma 3. If $d(v) = n - 3$, we assume $v_1, v_2 \notin N(v)$. Noting that $\alpha(G) = 2$, we have $v_1v_2 \in E(G)$ and hence G_0 contains a Hamilton cycle $C = v_1v_2 \cdots v_{n-1}$ such that $v_i \in N(v)$ for $3 \leq i \leq n - 1$. In this case, it is easy to see G contains T. If $d(v) \geq n - 2$, then since G_0 contains a Hamilton cycle, it is not difficult to see G contains T. $\square$

The following lemma is well known and can be found in many graph theory textbooks, see for instance [2].

Lemma 5. *A bipartite graph G with bipartition (U, V) contains a matching saturating U if and only if $|N(S)| \geq |S|$ for every $S \subseteq U$.*

3. Proof of Theorem 9

Proof of Theorem 9. Let T be a given tree of order $n \geq 5$. If $|L(T)| = 2$, then $T = P_n$ and hence Theorem 9 holds by Theorem 2. If $T \in \mathcal{T}$, then Theorem 9 holds by Theorems 3–5. Thus we may assume $|L(T)| \geq 3$ and $T \notin \mathcal{T}$.

We use induction on n. If $5 \leq n \leq 8$, then Theorem 9 holds by Theorem 7. In the following proof, we assume $n \geq 9$ and Theorem 9 holds for small values of n.

Let G be a graph of order $2n - 1$. If $\overline{G}$ contains no W_6, then $\alpha(G) \leq 6$. Let I be a maximum independent set of G. By Lemma 2, we may assume $|I| \geq 3$. Let $I = \{v_1, v_2, \ldots, v_k\}$, where $3 \leq k \leq 6$.

Suppose to the contrary G contains no T. We now consider the following two cases.

Case 1. $k = 3$.

In order to prove Case 1, we need the following three claims.

Claim 1. *G contains an induced subgraph $K_1 \cup K_2 \cup K_3$.*

Proof. Since G contains no T, by Theorem 8 we may assume T contains a II-deletable set U_0. By induction hypothesis, $G - I$ contains $T_{U_0} = T - U_0$. Let $N_T(U_0) = U$. If $|N_I(U)| \geq 2$, then G contains T, a contradiction. Hence $|N_I(U)| = 1$. This implies G has an induced subgraph $2K_1 \cup K_2$. Assume, without loss of generality, that $G[I_1] = 2K_1 \cup K_2$ with $I_1 = I \cup \{v_4\}$ and $v_3v_4 \in E(G)$. By induction hypothesis, $G - I_1$ contains T_{U_0}. If $|N_{I_1}(U)| \geq 2$, then G contains T, a contradiction. Hence $|N_{I_1}(U)| = 1$. Thus, noting that $k = 3$, we may assume $N_{I_1}(U) = \{v_2\}$. Let $I_2 = I_1 \cup U$. Since $k = 3$, it is easy to see that $G[I_2] = K_1 \cup K_2 \cup K_3$. □

In the following, we let $G_0 = K_1 \cup K_2 \cup K_3$ with $V(G_0) = X = \{x_1, x_2, \ldots, x_6\}$ and $E(G_0) = \{x_2x_3, x_4x_5, x_4x_6, x_5x_6\}$ be an induced subgraph of G.

Claim 2. $|L(T)| \geq 4$.

Proof. Let $L(T) = U_0$. If $|L(T)| = 3$, then by Theorem 8 we may assume U_0 is a IV-deletable set or III-deletable set. Let $N_T(U_0) = U$.

If $|U| = 2$, we assume $U = \{u_1, u_2\}$. In this case, it is easy to see $T_{U_0} = P_{n-3}$ and either $d_T(u_1) = 3$ or $d_T(u_2) = 3$. By Theorem 2, $G - I$ contains a P_{n-2}. Assume $P_{n-2} = p_1p_2 \cdots p_{n-2}$ to be a path in $G - I$. If $N_I(p_1) \cap N_I(p_{n-2}) \neq \emptyset$, then G contains a cycle C of length $n - 1$. Let $V = V(G) - V(C)$, then $d_V(v) = 0$ for any $v \in V(C)$ since otherwise G contains T. Thus we have $\alpha(G[V]) \leq 2$ since $\alpha(G) = 3$ and $\delta(G[V]) \geq n - 3$ by Lemma 1 and hence $G[V]$ contains T by Lemma 4. Thus we may assume $N_I(p_1) \cap N_I(p_{n-2}) = \emptyset$. In this case, if $d_I(p_1) \geq 2$ or $d_I(p_{n-2}) \geq 2$, then G contains T and hence we may assume $N_I(p_1) = \{v_1\}$ and $N_I(p_{n-2}) = \{v_2\}$. Let $V_0 = V(G) - I - P_{n-2}$, then $|V_0| = n - 2$. If $d_{V_0}(v_1) \geq 2$ or $d_{V_0}(v_2) \geq 2$, then G contains T. Thus, since $|V_0| = n - 2 \geq 7$, there are three vertices $w_1, w_2, w_3 \in V_0$ such that $v_iw_j \notin E(G)$ for $i = 1, 2$ and $j = 1, 2, 3$. If $N(p_2) \cap (V_0 \cup \{v_1\}) \neq \emptyset$ or

$N_{V_0}(p_{n-2}) \neq \emptyset$, then G contains T. Hence we have $N(p_2) \cap (V_0 \cup \{v_1\}) = N_{V_0}(p_{n-2}) = \emptyset$. Thus, $\overline{G}[v_1, p_2, v_2, p_{n-2}, w_1, w_2, w_3]$ contains a W_6 with the hub v_1, a contradiction.

If $|U| = 3$, we assume $U = \{u_1, u_2, u_3\}$. By induction hypothesis, $G - X$ contains T_{U_0}. Assume $d_X(u_1) \leq d_X(u_2) \leq d_X(u_3)$. Since G contains no T, by Lemma 5, we have $d_X(u_1) \leq d_X(u_2) \leq 2$. If $d_X(u_1) = 1$, then since $k = 3$, we have $N_X(u_1) = \{x_1\}$. If $x_1u_2 \in E(G)$, then since $d_X(u_2) \leq 2$, by the symmetry of x_2 and x_3, we may assume $x_2u_2 \notin E(G)$. Thus $\overline{G}[x_2, u_1, u_2, x_1, x_4, x_5, x_6]$ contains a W_6 with the hub x_2, a contradiction. If $x_1u_2 \notin E(G)$, then since $d_X(u_2) \leq 2$ and $k = 3$, we must have $N_I(u_2) = \{x_2, x_3\}$. In this case, $\overline{G}[x_1, u_2, x_2, x_3, x_4, x_5, x_6]$ contains a W_6 with the hub x_1, again a contradiction. Thus we have $d_X(u_1) = 2$. If $|N_X(U)| \geq 3$, then by Lemma 5, G contains T, a contradiction. Hence we have $|N_X(U)| = 2$ which implies $N_X(u_1) = N_X(u_2) = N_X(u_3)$. Let $W = V(G) - N_X(U) - V(T_{U_0})$, then $|W| = n$. Obviously, $d_W(u_i) = 0$ for $i = 1, 2, 3$ since otherwise G contains T. This implies $\alpha(G[W]) \leq 2$ since $k = 3$. And then $\delta(G[W]) \geq n - 3$ by Lemma 1. Hence $G[W]$ contains T by Lemma 4, a contradiction. □

Claim 3. *G contains no induced subgraph $3K_2$.*

Proof. Since G contains no T, by Theorem 8 we may assume T contains a III-deletable set or IV-deletable set U_0 and $N_T(U_0) = U$. Suppose to the contrary G contains an induced subgraph $G_1 = 3K_2$ with $V(G_1) = Y = \{y_i \mid 1 \leq i \leq 6\}$ and $E(G_1) = \{y_1y_2, y_3y_4, y_5y_6\}$. By induction hypothesis, $G - Y$ contains T_{U_0}. Since $k = 3$, it is easy to see $d_Y(u) \geq 2$ for any $u \in U$. If $|N_Y(U)| \geq 3$ and $|U| = 2$, then it is easy to see G contains T, a contradiction. If $|N_Y(U)| \geq 3$ and $|U| = 3$, then G contains T by Lemma 5, a contradiction. Thus we have $|N_Y(U)| = 2$. Since $k = 3$, we may assume $N_Y(U) = \{y_5, y_6\}$. In this case, G contains an induced subgraph $G_2 = 2K_2 \cup K_4$. Let $V(G_2) = Z = \{z_i \mid 1 \leq i \leq 8\}$ and $E(G_2) = \{z_1z_2, z_3z_4\} \cup \{z_iz_j \mid 5 \leq i < j \leq 8\}$. By Claim 2, we have $|L(T)| \geq 4$. By Theorem 8 we may assume T contains a IV-deletable set U_1. If $d_Z(u) \geq 4$ for any $u \in N_T(U_1)$, then G contains T, a contradiction. Hence there is some vertex $u_0 \in N_T(U_1)$ such that $d_Z(u_0) \leq 3$. Set $V = \{z_5, z_6, z_7, z_8\}$. Since $k = 3$, we have $d_V(u_0) \leq 1$. Hence we may assume $N_Z(u_0) \cap \{z_5, z_6, z_7\} = \emptyset$. Since $d_Z(u_0) \leq 3$, we may assume $z_1 \notin N_Z(u_0)$. Thus $\overline{G}[z_1, u_0, z_3, z_4, z_5, z_6, z_7]$ contains a W_6 with the hub z_1, a contradiction. □

In the following we prove Case 1.

By Theorem 8, T contains a III-deletable set or a IV-deletable set U_0. Let $N_T(U_0) = U$. By induction hypothesis, $G - X$ contains T_{U_0}. If there is some vertex $u \in U$ such that $d_X(u) = 1$, then since $k = 3$, we have $N_X(u) = \{x_1\}$ and thus G contains an induced subgraph $3K_2$ which contradicts Claim 3. Hence we have $d_X(u) \geq 2$ for any $u \in U$. If $|N_X(U)| \geq 3$ and $|U| = 2$, then G contains T, a contradiction. If $|N_X(U)| \geq 3$ and $|U| = 3$, then G contains T by Lemma 5, a contradiction. Hence we have $|N_X(U)| = 2$ which implies $N_X(u) = N_X(U)$ for each $u \in U$. If $x_1 \in N_X(U)$, then by the symmetry of x_2 and x_3 and Claim 3, we may assume $x_3 \in N_X(U)$ and hence $\overline{G}[x_2, u_1, u_2, x_1, x_4, x_5, x_6]$ contains a W_6 with the hub x_2, where $u_1, u_2 \in U$, a contradiction. If $x_1 \notin N_X(U)$, then since $k = 3$, we have $N_X(U) = \{x_2, x_3\}$ which implies $\overline{G}[x_1, u_1, x_2, x_3, x_4, x_5, x_6]$ contains a W_6 with the hub x_1, where $u_1 \in U$, a contradiction.

Case 2. $k \geq 4$.

If $k = 4$, then by Theorem 8 we may assume T contains a II-deletable set U_0. By induction hypothesis, $G - I$ contains T_{U_0}. Let $N_T(U_0) = U$. If $|N_I(U)| \geq 2$, then G contains T, a contradiction. Thus we have $|N_I(U)| = 1$ which implies G contains an induced subgraph $3K_1 \cup K_3$. Let $G' = 3K_1 \cup K_3$ with $V(G') = W = \{w_i \mid 1 \leq i \leq 6\}$ and $E(G') = \{w_4w_5, w_4w_6, w_5w_6\}$. By Theorem 8 we may assume T contains a III-deletable set U_1. Let $N_T(U_1) = U_2$. By induction hypothesis, $G - W$ contains T_{U_1}. If $d_W(u) \geq 3$ for each $u \in U_2$, then G contains T, a contradiction. Hence there is some vertex $u_0 \in U_2$ such that $d_W(u_0) \leq 2$. Since $k = 4$, we have $|N(u_0) \cap \{w_4, w_5, w_6\}| \leq 1$. Since $d_W(u_0) \leq 2$, we may assume $w_1 \notin N(u_0)$. Thus $\overline{G}[w_1, w_2, w_3, u_0, w_4, w_5, w_6]$ contains a W_6 with the hub w_1, a contradiction.

Let now $k = 5, 6$. By Theorem 8 we may assume T contains a 3-deletable set U_0. Let $N_T(U_0) = U$. By induction hypothesis, $G - I$ contains T_{U_0}. If $d_I(u) \geq 3$ for each $u \in U$, then G contains T, a contradiction. Hence there is some vertex $u \in U$ such that $d_I(u) \leq 2$. Thus, if $k = 5$, then G contains an induced subgraph $3K_1 \cup P_3$ or $4K_1 \cup K_2$. By an analogous argument of $k = 4$, we can get a contradiction. If $k = 6$, then $\overline{G}[I \cup \{u\}]$ contains a W_6, a contradiction.

From the proof above, we have $R(T, W_6) \leq 2n - 1$ for $T \notin \mathcal{T}$. On the other hand, the graph $2K_{n-1}$ shows $R(T, W_6) \geq 2n - 1$ for any tree T of order n and hence $R(T, W_6) = 2n - 1$ for $T \notin \mathcal{T}$. Thus the proof of Theorem 9 is completed. □

4. Proof of Theorem 10

Proof of Theorem 10. Let G be a graph of order $3n - 2$ and T a given tree of order n. Suppose $\overline{G}$ contains no W_7.

Claim 4. *If G contains no T, then $\delta(G) = n - 2$.*

Proof. By Theorem 3, we may assume $T \neq S_n$. Let $d(v) = \delta(G)$ and $V = V(G) - N[v]$. If $\delta(G) \leq n - 3$, then $|V| \geq 2n$. Since G contains no T, by Theorem 9, $\overline{G}[V]$ contains a W_6 and hence $\overline{G}[V]$ contains a C_7 which implies $\overline{G}$ contains a W_7 with the hub v, a contradiction. Hence we have $\delta(G) \geq n - 2$. If $\delta(G) \geq n - 1$, then it is easy to see G contains all trees of order n. Thus we have $\delta(G) \leq n - 2$ and hence $\delta(G) = n - 2$. □

By Theorem 3, G contains a tree $T_* = S_n$. Let $V(T_*) = \{v_0, v_1, \ldots, v_{n-1}\}$ and $E(T_*) = \{v_0v_i \mid 1 \leq i \leq n - 1\}$. If G contains no $S_n(1, 1)$, then by Claim 4, we have $d(v_1) \geq n - 2 \geq 4$ and hence there is some vertex $w \in V(G)$ such that $w \neq v_0$ and $w \in N(v_1)$ which implies G contains an $S_n(1, 1)$, a contradiction. Assume $T_{**} = S_n(1, 1)$ with $V(T_{**}) = \{u_0, \ldots, u_{n-1}\}$ and $E(T_{**}) = \{u_0u_i \mid 1 \leq i \leq n - 2\} \cup \{u_1u_{n-1}\}$. If G contains no $S_n(1, 2)$, then by Claim 4, we have $d(u_{n-1}) \geq n - 2 \geq 4$ and hence there is some vertex $w \in V(G)$ such that $u \neq u_0, u_1$ and $w \in N(u_{n-1})$ which implies G contains an $S_n(1, 2)$, a contradiction. Thus we may assume $T \neq S_n$, $S_n(1, 1)$ and $S_n(1, 2)$. Assume $d(v) = \delta(G)$ and $V = V(G) - N[v]$. If G contains no T, then by Claim 4, we have $|V| = 2n - 1$. By Theorem 9, $\overline{G}[V]$ contains a W_6 and hence $\overline{G}[V]$ contains a C_7 which implies $\overline{G}$ contains a W_7 with the hub v, a contradiction. Thus we have

$R(T, W_7) \leq 3n - 2$. On the other hand, the graph $3K_{n-1}$ shows $R(T, W_7) \geq 3n - 2$ and hence $R(T, W_7) = 3n - 2$, that is, $R(T_n, W_7) = 3n - 2$. The proof of Theorem 10 is completed. □

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