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A NOTE ON ARC PANCYCLICITY OF REGULAR ORDINARY MULTIPARTITE TOURNAMENTS*

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Abstract In this paper, we prove that regular ordinary m-partite ($m \geqslant 5$) tournaments are arc pancyclic. Our result is a generalization of Alspach's theorem [1] for regular tournaments.

Keywords tournaments, ordinary multipartite tournaments, arc pancyclicity.
AMS(1991) subject classifications 05C03, 05C20.

1 Introduction

We use the terminology and notation of [4]. Let $D=(V(D),A(D))$ be a digraph. If xy is an arc of a digraph D, then we say that x dominates y, denoted by $x \rightarrow y$. The outset $N^+(x)$ of a vertex x is the set of vertices dominated by x, and the inset $N^-(x)$ is the set of vertices dominating x. A digraph D is said to be regular if there is an integer r such that $|N^+(x)|=|N^-(x)|=r$ holds for every $x \in V(D)$. By a cycle (path) we mean a directed cycle (directed path). A cycle of length m is called an m-cycle. A digraph D is arc pancyclic if every arc of D is contained in an m-cycle for all m between 3 and $|V(D)|$ inclusive. We shall use $x^+(x^-)$ to denote the successor (predecessor) of x on a path or a cycle. If C is a cycle and u, v are two vertices of C, then we use $C[u,v]$ to denote the subpath of C from u to v.

It is well known that tournaments have a very rich structure. Recently it has been shown that there are several classes of much more general digraphs containing tournaments, which share many properties of tournaments. In [2] a generalization of tournaments is introduced as follows: an ordinary multipartite tournament results from a tournament by substituting each vertex with an independent vertex set. [5] characterizes pancyclic and vertex pancyclic ordi-

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nary multipartite tournaments and [3] characterizes weakly Hamilton-connected ordinary multipartite tournaments. Obviously, a tournament is an ordinary multipartite tournament. Let T be an ordinary multipartite tournament and $x \in V(T)$, we denote $V^c(x)$ the partite set of T to which x belongs.

B. Alspach[1] shows that every regular tournament is arc pancyclic. In this paper we prove the following theorem which is a gneralization of Alspach's theorem.

Theorem Every regular ordinary m-partite($m \geqslant 5$) tournament is arc pancyclic.

Note that there is no 4-cycle in regular ordinary 3-partite tournaments. So the above theorem is not true for $m=3$. By Lemma 1 as below, it is impossile for $m=4$.

In Figure 1, we give an example of almost regular ordinary 5-partite tournament with 9 vertices, in which arc e is not in a 9-cycle. So our result is the best possible in a sense.

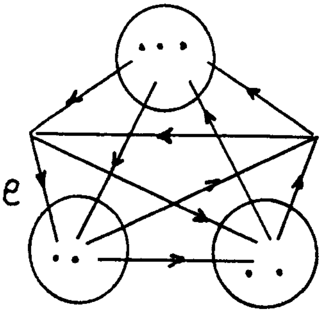

Figure 1

2 Proof of Theorem

Let T be a regular ordinary m-partite ($m \geqslant 5$) tournament with partite sets $V_1, V_2, \cdots, V_m$ and $e=(v_1, v_2)$ be an are of T. Without loss of generality, suppose $v_1 \in V_1$, $v_2 \in V_2$.

Lemma 1 $|V_i|=|V_j|$, for $1 \leqslant i \neq j \leqslant m$.

Proof Let $v_i \in V_i$, $v_j \in V_j$. $|V(T)|-|V_i|=d^+(v_i)+d^-(v_i)=d^+(v_j)+d^-(v_j)=|V(T)|-|V_j|$ since T is regular. So $|V_i|=|V_j|$.

By Lemma 1, we may denote $|V_i|=k$, $1 \leqslant k \leqslant m$.

Lemma 2 Let $v_i \in V_i, 1 \leqslant i \leqslant m$, then $T'=T\langle v_1, v_2, \cdots, v_m\rangle$ is a regular tournament.

Proof By the definition of regular ordinary multipartite tournaments and Lemma 1, it is easy to obtain that $d^+_{T'}(v_i)=d^-_{T'}(v_i)=\frac{m-1}{2}$.

Lemma 3(Moon [6]) Every strong tournament is vertex pancyclic.

Proof of Theorem We take vertex $v_i \in V_i, 3 \leqslant i \leqslant m$. By Lemma 2, $T_1=T\langle v_1, v_2, \cdots, v_m\rangle$ is a regular tournament. Clearly $T-V(T_1)$ is also regular. Let $v_{i2} \in V_i \backslash v_i$, $1 \leqslant i \leqslant m$. $T_2=T\langle v_{12}, v_{22}, \cdots, v_{m2}\rangle$ is a regular tournament. Along this way, we obtain $T_1, T_2, \cdots, T_k$ such that $V(T)=V(T_1) \cup V(T_2) \cup \cdots \cup V(T_k)$, $V(T_i) \cap V(T_j) \neq \varnothing$ for $1 \leqslant i \neq j \leqslant k$, and T_i is a regular tournament and has exactly one vertex from each partite set for $1 \leqslant i \leqslant k$. Without loss of generality, let $e \in A(T_1)$.

Clearly T_1 has an l-cycle which includes e for $3\leqslant l\leqslant m$ since T_1 is a regular tournament. Let C_1 be an n-cycle in T_1, $3\leqslant n\leqslant m$. For a given l, $3\leqslant l\leqslant m$, now we will prove that T has an $(n+l)$-cycle which includes e. Let $v\in V(T_2)$ such that $V^c(v_2)=V^c(v)$. By Lemma 3, T_2 has an l-cycle C_2 which contains v. For cycles C_1 and C_2, we have $v\rightarrow v_2^+$ and $v_2\rightarrow v^+$ in T. So there is an $(n+l)$-cycle in T $v_1v_2C_2[v^+,v]C_1[v_2^+,v_1]$, which contains arc e. Similarly, we can prove that T has an l-cycle which contains arc e, for $3\leqslant l\leqslant |V(T)|$. This completes the proof of the theorem.

Stimulated by [7], we give the following conjecture.

Conjecture Let T be an ordinary m-partite ($m\geqslant 4$) tournament. If every arc of T on a 3-cycle, then T is arc pancyclic except for some special classes.

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正则一致多部竞赛图的弧泛圈性的注记

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摘要 本文证明了正则一致 m-部($m\geqslant 5$)竞赛图是弧泛圈的，从而推广了文[1]中关于正则竞赛图的 Alspach 的定理．

关键词 竞赛图，一致多部竞赛图，弧泛圈性

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