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THE RAMSEY NUMBERS FOR STARS AND STRIPES*

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Abstract Let $\Sigma = \sum_{i=1}^{t}(n_i-1)$ and $\Lambda = \sum_{j=1}^{s}(m_j-1)$. This paper considers the generalized Ramsey number $R(K_{1,n_1},\cdots,K_{1,n_t},m_1K_2,\cdots,m_sK_2)$ for any Σ and Λ. And the authors get their exact values if $1 \leqslant \Lambda \leqslant \Sigma$ and their upper bounds if $\Lambda > \Sigma$.

Key words Ramsey number; Stars; Stripes.

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1 Introduction and Lemmas

All graphs will be finite and undirected without loops or multiple edges. All undefined terms see [2]. $\beta(G)$ is denoted the number of edges in the maximum matching of graph G. Let $\Sigma = \sum_{i=1}^{t}(n_i-1)$ and $\Lambda = \sum_{j=1}^{s}(m_j-1)$, where m_i, n_i are positive integers. Let $G_1, G_2, \cdots, G_m$ be simple graphs. The generalized Ramsey number $R(G_1, G_2, \cdots, G_m)$ is the smallest integer n such that every m-edge coloring $(E_1, E_2, \cdots, E_m)$ of K_n contains, for some i, a subgraph isomorphic to G_i in color i. The problem of the generalized Ramsey number about the stars or stripes is interesting for many people such as [1], [3], [5] and [6].

Theorem A[1] (i) If Σ is odd, then $R(K_{1,n_1}, \cdots, K_{1,n_t}) = \Sigma + 2$;

(ii) If Σ is even and all n_i are odd, then $R(K_{1,n_1}, \cdots, K_{1,n_t}) = \Sigma + 2$;

(iii) If Σ is even and at least one n_i is even, then $R(K_{1,n_1}, \cdots, K_{1,n_t}) = \Sigma + 1$.

Theorem B[3] Let $m_1, m_2, \cdots, m_s$ be integers and $m_1 = \max\{m_1, m_2, \cdots, m_s\}$. Then $R(m_1K_2, m_2K_2, \cdots, m_sK_2) = m_1 + 1 + \Lambda$.

In this paper, we consider the generalized form such as $R(K_{1,n_1}, \cdots, K_{1,n_t}, m_1K_2, \cdots, m_sK_2)$. For this purpose, we need the following Lemmas:

Lemma 1[4] Let G be a connected graph with $|V(G)| > 2\delta(G)$, then G contains a path with length $\geqslant 2\delta(G)$.

Lemma 2[4] Let G be a 2-connected graph with $|V(G)| \geqslant 2\delta(G)$, then G contains a cycle with length $\geqslant 2\delta(G)$.

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Lemma 3 Let G be a connected graph, then $\beta(G) \geqslant \min\{|V(G)|/2, \delta(G)\}$.

Proof If $\delta(G) \geqslant |V(G)|/2$, thus G contains a Hamilton cycle. Hence $\beta(G) = [|V(G)|/2]$. If $\delta(G) < |V(G)|/2$, thus $|V(G)| > 2\delta(G)$. Hence $\beta(G) \geqslant \delta(G)$ by Lemma 1.

2 Theorems and Their Proofs

Theorem 1 Let $R =: R(K_{1,n_1}, \cdots, K_{1,n_t}, mK_2)$, thus

$$R = \begin{cases} 2m & \text{if } m \geqslant \Sigma + 1; \\ m + \Sigma + 1 & \text{if } m < \Sigma + 1 \text{ and } 2 \nmid \Sigma \text{ or } 2|\Sigma \text{ and all of } n_i \text{ are odd;} \\ m + \Sigma & \text{if } m < \Sigma + 1 \text{ and } 2|\Sigma \text{ and at least one of } n_i \text{ is even.} \end{cases}$$

Proof (1) $m \geqslant \Sigma + 1$. Let all of edges of K_{2m-1} have color α_{t+1}. Thus there is neither mK_2 in color α_{t+1} nor K_{1,n_i} $(i = 1, 2, \cdots, t)$ in color α_i on K_{2m-1}. Hence we have $R \geqslant 2m$. On the other hand, let K_{2m} be colored by colors $\alpha_1, \alpha_2, \cdots, \alpha_{t+1}$. If there is no K_{1,n_i} in color α_i $(i = 1, 2, \cdots, t)$, we consider an edge induced subgraph H_1 by all of edges in color α_{t+1}. Clearly, $\delta(H_1) \geqslant 2m - 1 - \Sigma \geqslant m$. Thus H_1 has a Hamilton cycle. So $\beta(H_1) = m$. Hence $R \leqslant 2m$. Therefore $R = 2m$ if $m \geqslant \Sigma + 1$.

(2) $m < \Sigma + 1$ and Σ is odd or Σ is even and all of n_i are odd. Since Theorem A(i) and (ii), $G = K_{\Sigma+1} \cup K_{m-1}$ can be colored by colors $\alpha_1, \alpha_2, \cdots, \alpha_t$ such that G does not contain K_{1,n_i} in color α_i $(i = 1, 2, \cdots, t)$. And G^c is colored by color α_{t+1}. It is easy to get $\beta(G^c) = m - 1 < m$. Hence $R \geqslant m + \Sigma + 1$.

On the other hand, let $K_{m+\Sigma+1}$ be colored by colors $\alpha_1, \alpha_2, \cdots, \alpha_{t+1}$. If there is no K_{1,n_i} in color α_i $(i = 1, 2, \cdots, t)$, we consider the edges induced subgraph H_2 by all of edges in color α_{t+1}. Clearly, $\delta(H_2) \geqslant (m + \Sigma) - \Sigma = m$. If H_2 is connected, by Lemma 3, $\beta(H_2) \geqslant \min\{[(m + \Sigma + 1)/2], m\} = m$. If H_2 isn't connected, thus let C_1, C_2 be two components of H_2. Since $\delta(C_i) \geqslant \delta(H_2) \geqslant m$ $(i = 1, 2)$. $\beta(H_2) \geqslant \beta(C_1) + \beta(C_2) \geqslant \min\{[|V(C_1)|/2], \delta(C_1)\} + \min\{[|V(C_2)|/2], \delta(C_2)\} \geqslant 2\min\{[(m + 1)/2], m\} \geqslant m$ by Lemma 3. Hence $R \leqslant m + \Sigma + 1$. Therefore $R = m + \Sigma + 1$ if $m < \Sigma + 1$ and Σ is odd or Σ is even and all of n_i are odd.

(3) $m < \Sigma + 1$ and Σ is even and at least one of n_i is even. By Theorem A(iii), using an analogous to the proof of (2), we can get $R \geqslant m + \Sigma$.

Now we prove the reverse inequality. Let the edges of $K_{m+\Sigma}$ be colored by colors $\alpha_1, \alpha_2, \cdots, \alpha_{t+1}$. If there is no edges of K_{1,n_i} in color α_i $(i = 1, 2, \cdots, t)$, we consider the edge induced subgraph H_3 by all of edges in color α_{t+1}. Thus $\delta(H_3) \geqslant (m + \Sigma - 1) - \Sigma = m - 1$.

If H_3 has at least three components, say C_1, C_2, C_3. Thus we have $\delta(C_i) \geqslant \delta(H_3)$ $(i = 1, 2, 3)$. For $m = 1, 2, 3$, it is easy to get $\beta(H_3) \geqslant m$. For $m \geqslant 4$, by Lemma 3, $\beta(H_3) \geqslant \beta(C_1) + \beta(C_2) + \beta(C_3) \geqslant \sum\limits_{i=1}^{3} \min\{[|V(C_i)|/2], \delta(C_i)\} \geqslant 3\min\{[m/2], m-1\} = 3[m/2] \geqslant m$. If H_3 has exactly two components C_1, C_2, thus $|V(C_1)| \geqslant m, |V(C_2)| \geqslant m$. If $|V(C_1)| = |V(C_2)| = m$, then $m = \Sigma =$ even and C_1, C_2 are complete graphs. Hence $\beta(H_3) = m/2 + m/2 = m$.

If $\max\{|V(C_1)|, |V(C_2)|\} \geqslant m + 1$, say $|V(C_1)| \geqslant m + 1$. By Lemma 3, we have $\beta(H_3) = \beta(C_1) + \beta(C_2) \geqslant \sum\limits_{i=1}^{2} \min\{[|V(C_i)|/2], \delta(C_i)\} \geqslant \min\{[(m+1)/2], m - 1\} + \min\{[m/2], m - 1\} = [(m + 1)/2] + [m/2] = m$. If H_3 is connected with a cut vertex v and if $H_3 - v$ has at least three components, say D_1, D_2, D_3. For $m = 1,2,3,4$, it is easy to get $\beta(H_3) \geqslant m$. For $m \geqslant 5$, by Lemma 3, $\beta(H_3) \geqslant \beta(D_1) + \beta(D_2) + \beta(D_3) \geqslant \sum_{i=1}^{3} \min\{[|V(D_i)|/2], \delta(D_i)\} \geqslant 3\min\{[(m-1)/2], m-2\} = 3[(m-1)/2] \geqslant m$. If $H_3 - v$ has exactly two components, say D_1, D_2, thus we can assume that $|(V(D_1)| \geqslant m, |V(D_2)| \geqslant m-1$ and $|V(D_1)| \geqslant |V(D_2)|$ since $|V(H_3 - v)| = m + \Sigma - 1 \geqslant 2m-1, \delta(D_1) \geqslant m-2$ and $\delta(D_2) \geqslant m-2$. It is easy to prove $\beta(H_3) \geqslant m$ if $m \leqslant 3$. If $\Sigma = m$, then $|V(D_1)| = m$ and $|V(D_2)| = m-1$. For $m \geqslant 4$, there are Hamilton cycles in D_1 and D_2 respectively. Since $N(v) \cap V(D_2) \neq \emptyset$ and $m = \Sigma =$ even, $\beta(H_3) = \beta(D_1) + \beta(H_3[V(D_2) \cup \{v\}]) = m/2 + m/2 = m$. If $\Sigma = m+1$, thus m is odd. i.e., $m \geqslant 5$. Using an analogous method as above, we can get $\beta(H_3) \geqslant m$. If $\Sigma \geqslant m+2$ and $m \geqslant 4$, thus $|V(D_1)| + |V(D_2)| = \Sigma + m - 1 \geqslant 2m+1$. Hence we always have $[|V(D_1)|/2] + [|V(D_2)|/2] \geqslant m$. Note that $\delta(D_1) + \delta(D_2) \geqslant 2(m-2) \geqslant m, \delta(D_1) + [|V(D_2)|/2] \geqslant m - 2 + [(m-1)/2] \geqslant m$ if $m \geqslant 5$ and $[|V(D_1)|/2] + \delta(D_2) \geqslant [m/2] + m - 2 \geqslant m$. Therefore, by Lemma 3, $\beta(H_3) \geqslant \beta(D_1) + \beta(D_2) = \sum_{i=1}^{2} \min\{[|V(D_i)|/2], \delta(D_i)\} \geqslant m$ if $m \geqslant 5$ or $m = 4$ and $|V(D_2)| \geqslant m$. Hence the remaining case is $D_2 = K_3$. At this time, we have $\beta(H_3) = \beta(D_1) + \beta(H_3[V(D_2) \cup \{v\}]) \geqslant m$.

If H_3 is 2-connected and if $\delta(H_3) \geqslant (m+\Sigma)/2$, thus there is a hamilton cycle in H_3. Hence $\beta(H_3) \geqslant m$. If $\delta(H_3) < (m+\Sigma)/2$, then $m + \Sigma > 2\delta(H_3) \geqslant 2(m-1)$. By Lemma 2, there is a cycle C in H_3 with length $\geqslant 2(m-1)$. If there is a cycle in H_3 with length $\geqslant 2m$, then $\beta(H_3) \geqslant m$. If there is a cycle C in H_3 with length $2m-1$, thus there is a vertex $x \notin C$ which is adjacent with C. So $\beta(H_3) \geqslant m$. If the length of the longest cycle C is $2m-2$, say $C = (x_1, x_2, \cdots, x_{2m-2})$, then $\beta(H_3) \geqslant m$. In fact, otherwise $\beta(H_3) = m-1$, thus $V(H_3) - V(C) = \{y_1, y_2, \cdots, y_{\Sigma-m+2}\}$ is an independent set of H_3. Since C is a longest cycle in H_3 and $\delta(H_3) \geqslant m-1$, we can assume that $N(y_1) = N(y_2) = \cdots = N(y_{\Sigma-m+2}) = \{x_1, x_3, \cdots, x_{2m-3}\}$. And then $\{x_2, x_4, \cdots, x_{2m-2}\} \cup (V(H_3) - V(C))$ is an independent set of H_3 with size $(\Sigma - m + 2) + (m-1) = \Sigma + 1$. Hence, by Theorem A(iii) on $K_{m+\Sigma}$, there is a subgraph K_{1,n_i} in color α_i, a contradiction.

Theorem 2 If $\Lambda < \Sigma$, then

$$R =: R(K_{1,n_1}, K_{1,n_2}, \cdots, K_{1,n_t}, m_1K_2, m_2K_2, \cdots, m_sK_2)$$
$$= \begin{cases} \Lambda + \Sigma + 2 & \text{if } \Sigma \text{ is odd or } \Sigma \text{ is even and all of } n_i \text{ are odd,} \\ \Lambda + \Sigma + 1 & \text{if } \Sigma \text{ is even and at least one of } n_i \text{ is even.} \end{cases}$$

Proof (1) Σ is odd or Σ is even and all of n_i are odd. Since Theorem A(i) and (ii), $G = K_{\Sigma+1} \cup K_\Lambda$ can be colored by colors $\alpha_1, \alpha_2, \cdots, \alpha_t$ such that G does not contain K_{1,n_i} in color α_t $(i = 1,2,\cdots,t)$. And then let $G^c = (X;Y;E)$ be a complete bipartite graph, where $|X| = \Lambda$ and $|Y| = \Sigma + 1$. Let $(X_1, X_2, \cdots, X_s)$ be a partition of X with $|X_j| = m_j - 1$ $(j = 1,2,\cdots,s)$. And let the edge of E, which is incident with a vertex in X_j, be colored by colors α_{t+j} $(j = 1,2,\cdots,s)$. Clearly G^c does not contain a subgraph m_jK_2 in color α_{t+j} $(j = 1,2,\cdots,s)$. Hence $R \geqslant \Lambda + \Sigma + 2$.

Now we prove the reverse inequality. Let $K_{\Lambda+\Sigma+2}$ be colored by colors $\alpha_1, \alpha_2, \cdots, \alpha_{t+s}$. If there is no K_{1,n_i} in color α_i $(i = 1,2,\cdots,t)$. By Theorem 1, there exists a $(\Lambda+1)K_2 (\subseteq K_{\Lambda+\Sigma+2})$ such that which edges in colors α_{t+j} $(j = 1,2,\cdots,s)$. So there is some m_jK_2 $(1 \leqslant j \leqslant s)$ in color α_{t+j}. Hence $R \leqslant \Lambda + \Sigma + 2$. i.e., $R = \Lambda + \Sigma + 2$.

(2) Σ is even and at least one of n_i is even. Using an analogous method of (1), we can get $R = \Lambda + \Sigma + 1$. #

Theorem 3 If $\Lambda \geqslant \Sigma$, and let $m_1 = \max\{m_1, m_2, \cdots, m_s\}$, $R =: R(K_{1,n_1}, \cdots, K_{1,n_t}, m_1K_2, \cdots, m_sK_2)$, we have:

(i) Σ is odd or Σ is even and all of n_i are odd , thus $\max\{m_1 + \Lambda + 1, \Lambda + \Sigma + 2\} \leqslant R \leqslant 2(\Lambda + 1)$;

(ii) Σ is even and at least one of n_i is even, then $\max\{m_1+\Lambda+1, \Lambda+\Sigma+1\} \leqslant R \leqslant 2(\Lambda+1)$.

Proof When Σ is odd, let $K_{2\Lambda+2}$ be colored by colors $\alpha_1, \alpha_2, \cdots, \alpha_{t+s}$. If there is no K_{1,n_i} in color α_i $(i = 1, 2, \cdots, t)$. By Theorem 1, there exists a $(\Lambda + 1)K_2(\subseteq K_{2\Lambda+2})$ whose edges is colored by color α_{t+j} $(j = 1, 2, \cdots, s)$. So, there is some m_jK_2 $(1 \leqslant j \leqslant s)$ in color α_{t+j}. Hence $R \leqslant 2(\Lambda + 1)$. By Theorem B, we have $R \geqslant m_1 + \Lambda + 1$. Hence in the following, we will prove that $R \geqslant \Lambda + \Sigma + 2$. Let $G = K_{\Lambda+\Sigma+1}$ and let $(V_1, V_2, \cdots, V_s, V_{s+1})$ be a partition on $V(G)$ with $|V_i| = m_i - 1$ $(i = 1, 2, \cdots, s)$ and $|V_{s+1}| = \Sigma + 1$. G is colored by colors $\alpha_1, \alpha_2, \cdots, \alpha_{t+s}$ as follows: let $e = xy \in G$. (1) $x, y \in V_1$, thus e is colored by color α_1; (2) $x \in V_i, y \in V_j$ and $i \leqslant j$, thus e is colored by color α_j $(j = 2, 3, \cdots, s)$; (3) $x \in V_i$ $(i = 1, 2, \cdots, s)$ and $y \in V_{s+1}$, thus e is colored by color α_i; (4) by Theorem A(i) $K_{|V_{s+1}|}$ can be colored by colors $\alpha_{s+1}, \cdots, \alpha_{s+t}$ such that there is no K_{1,n_i} in color α_{s+i} $(i = 1, 2, \cdots, t)$. Clearly, G is no m_jK_2 $(j = 1, 2, \cdots, s)$ in color α_j. Hence $R \geqslant \Lambda + \Sigma + 2$.

Using an analogous method as above, we can prove the remains part of (i) and (ii).

Theorem 4 If $\Lambda = \Sigma$, and $R =: R(K_{1,n_1}, \cdots, K_{1,n_t}, m_1K_2, \cdots, m_sK_2)$, $m_j \geqslant 2$ $(j = 1, 2, \cdots, s)$, then

$$R = \begin{cases} \Lambda + 2 & \text{If } s = 1 \text{ or } \Sigma \text{ is odd or } \Sigma \text{ is even and all of } n_i \text{ are odd; or} \\ & \text{if } 2|\Sigma \text{ and at least one } n_i \text{ is even and } s = 2 \text{ with } m_1 = m_2 \\ 2\Lambda + 1 & \text{If } 2|\Sigma \text{ and at least one } n_i \text{ is even and } s = 2 \text{ with } m_1 \neq m_2 \\ & \text{or } s \geqslant 3. \end{cases}$$

Proof By Theorem 1, we get $R = 2\Lambda + 2$ if $s = 1$. By Theorem 3(i), we get $R = 2\Lambda + 2$ if Σ is odd or Σ is even and all of n_i are odd.

Now, we consider the case that Σ is even and at least one n_i is even and $s = 2$ with $m_1 = m_2$. By Theorem 3, we have $R \leqslant 2\Lambda + 2$. On the other hand, let $V(K_{2\Lambda+1}) = \{x_1, x_2, \cdots, x_\Lambda, y_1, y_2, \cdots, y_\Lambda, z\}$, $X = \{x_1, x_2, \cdots, x_\Lambda\}$ and $Y = \{y_1, y_2, \cdots, y_\Lambda\}$. Clearly, $G = K_{|X|,|Y|}$ is 1-factorable. $(F_1, F_2, \cdots, F_t)$ is a partition of these Λ 1-factors with $|F_i| = n_i - 1$. All of edges in F_i are colored by color α_i $(i = 1, 2, \cdots, t)$, every edges of complete graph on $X \cup \{z\}$ is colored by color α_{t+1}, and every edge of the complete graph on $Y \cup \{z\}$ is colored by color α_{t+2}. Thus there is no K_{1,n_i} in color α_i $(i = 1, 2, \cdots, t)$ on $K_{2\Lambda+1}$, and there is also no m_jK_2 in color α_{t+j} $(j = 1, 2)$ on $K_{2\Lambda+1}$. Thus $R \geqslant 2\Lambda + 2$. Therefore $R = 2\Lambda + 2$ if Σ is even and $s = 2$ with $m_1 = m_2$.

In the following, we prove that $R = 2\Lambda + 1$ if Σ is even and at least one n_i is even and $s \geqslant 3$ or $s = 2$ with $m_1 \neq m_2$. By Theorem 3, we get $R \geqslant 2\Lambda + 1$. On the other hand, let $K_{2\Lambda+1}$ be colored by colors $\alpha_1, \alpha_2, \cdots, \alpha_{t+s}$. If there is no K_{1,n_i} in color α_i $(i = 1, 2, \cdots, t)$, and let H be an edge induced subgraph by all of edges which colored by colors $\alpha_{t+1}, \alpha_{t+2}, \cdots, \alpha_{t+s}$, thus $\delta(H) \geqslant 2\Lambda - \Sigma = \Lambda$. Clearly, H is connected, otherwise at least one component C with

$\delta(C) < \Lambda$, a contradiction. If there is a cut vertex v of H, then $H-v$ has exactly two components D_1, D_2 with $H[V(D_1) \cup \{v\}] = H[V(D_2) \cup \{v\}] = K_{\Lambda+1}$. Let C_1, C_2 be Hamilton cycles in D_1, D_2 respectively. For every j, let E_j be the set of edges with color α_{t+j} in $C_1 \cup C_2$. And let $a_j = |E_j|$, the edge induced subgraph $H_j = (C_1 \cup C_2)[E_j]$. Thus we have $\beta_j =: \beta(H_j) \geqslant \{a_j/2\}$. If there is an odd number a_j, then there is another odd number a_k since $C_1 \cup C_2$ has $2\Lambda+2$ edges. Thus we have $\sum_{j=1}^{s} \beta_j \geqslant \sum_{j=1}^{s} \{a_j/2\} \geqslant (\sum_{j=1}^{s}(a_j/2)+1) = \Lambda+1$. So there is some $m_{j_0}K_2$ $(1 \leqslant j_0 \leqslant s)$ in color α_{t+j_0}. Hence in the following we always assume that a_j $(1 \leqslant j \leqslant s)$ are even. Using an analogous method as above, we can get that every component of H_j $(1 \leqslant j \leqslant s)$ has even number of edges. Thus we have $\beta_j \geqslant a_j/2$ for every j $(1 \leqslant j \leqslant s)$, and then $\sum_{j=1}^{s} \beta_j \geqslant \sum_{j=1}^{s} a_j/2 = \Lambda$. If only one color, say α_{t+1}, appears on $C_1 \cup C_2$, then there is m_1K_2 with color α_{t+1} in H. If only two colors, say $\alpha_{t+1}, \alpha_{t+2}$, appear on $C_1 \cup C_2$, thus when $s \geqslant 3$ there is m_1K_2 in color α_{t+1} or m_2K_2 in color α_{t+2} since $\beta(C_1 \cup C_2) = \Lambda \geqslant (m_1 - 1) + (m_2 - 1) + 1$. Hence we only need to consider the case that at least three colors appear on $C_1 \cup C_2$. When $s = 2$, note that $m_1 \neq m_2$. So we can assume that there are at least two colors appear on C_1. Let v_1 be a common vertex of two monochromatic paths on C_1. Since v_1v must be colored by one of color α_{t+j} $(1 \leqslant j \leqslant s)$, there always exists some $m_{j_0}K_2$ in color α_{t+j_0} $(1 \leqslant j_0 \leqslant s)$.

If H is 2-connected, by Lemma 2, then H contains a cycle with length $\geqslant 2\Lambda$. If there is a cycle with length $2\Lambda + 1$, thus it always contains a monochromatic odd component in this cycle. So it is easy to see that there is a m_jK_2 $(1 \leqslant j \leqslant s)$ in color α_{t+j}. If the length of the longest cycle C is 2Λ in H. Let $V(H) - V(C) = \{u\}$. Clearly, since $d_H(u) \geqslant \delta(H) \geqslant \Lambda$, $d_H(u) = \Lambda$. In this case, if there is no m_jK_2 in color α_{t+j} for any $j \in \{1, 2, \cdots, s\}$, we can prove as above that every component of H_j in C is even. Let $C = (x_1, x_2, \cdots, x_{2\Lambda}, x_1)$, and let $N_H(u) = \{x_1, x_3, \cdots, x_{2\Lambda-1}\}$. Clearly, $V(C) - N_H(u)$ contains all of the common vertices of components of H_j $(1 \leqslant j \leqslant s)$. If there is an edge $x_{2i}x_{2j} \in E(H)$ $(1 \leqslant i < j \leqslant \Lambda)$, then there is a $(2\Lambda + 1)$-cycle in H, a contradiction. Hence $\{u, x_2, x_4, \cdots, x_{2\Lambda}\}$ is an independent set in H. By Theorem A (iii), there exists a K_{1,n_i} in color α_i $(1 \leqslant i \leqslant t)$. Combining all of these cases, we have $R \leqslant 2\Lambda + 1$. So $R = 2\Lambda + 1$ if Σ is even and at least one n_i is even and $s \geqslant 3$ or $s = 2$ with $m_1 \neq m_2$. This completes the proof of Theorem 4.

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关于星和条的 Ramsey 数

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摘要: 令 $\Sigma=\sum\limits_{i=1}^{t}(n_i-1)$ 和 $\Lambda=\sum\limits_{j=1}^{s}(m_j-1)$. 该文研究了广义 Ramsey 数 $R(K_{1,n_1},\cdots,K_{1,n_t},m_1K_2,\cdots,m_sK_2)$. 当 $1\leqslant\Lambda\leqslant\Sigma$ 时, 得到了它们的精确值; 当 $\Sigma>\Lambda$ 时, 得到了它们的上界.

关键词: Ramsey 数; 星; 条.
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