

**NECESSARY AND SUFFICIENT CONDITION  
FOR TWO LINEAR DIOPHANTINE EQUATIONS  
TO HAVE A COMMON SOLUTION**

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ABSTRACT

In this paper it is shown by induction that the system of linear Diophantine equations

$$\begin{cases} a_1x_1 + \cdots + a_kx_k = b \\ c_1x_1 + \cdots + c_kx_k = d \end{cases}$$

is solvable (over the integers) if and only if  $(a_1, \cdots, a_k) \mid b$ ,  $(c_1, \cdots, c_k) \mid d$  and

$$\left( \left| \begin{array}{cc} a_i & a_1 \\ c_i & c_1 \end{array} \right|, \cdots, \left| \begin{array}{cc} a_i & a_k \\ c_i & c_k \end{array} \right| \right) \mid \left| \begin{array}{cc} a_i & b \\ c_i & d \end{array} \right|$$

for every  $i = 1, \cdots, k$ . (The notation  $(n_1, \cdots, n_k)$  denotes the greatest common divisor of  $n_1, \cdots, n_k$ .)