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SYSTEMS OF CONGRUENCES WITH MULTIPLIERS

SUN Zhiwei

(Nanjing University)

Abstract

The covering systems of congruences have been investigated by many authors. The purpose of this paper is to study the systems of congruences with multipliers which have the form $\{(\lambda_s, a_s, n_s)\}_{s=1}^k$ where $0 \leqslant a_s < n_s$ $(a_s, n_s \in \mathbb{Z})$ and $\lambda_s \in M$. (M is a commutative monoid considered as an additive one.) In Sec. 2 we develop a method to generate all those $A = \{(\lambda_s, a_s, n_s)\}_{s=1}^k$ with $\lambda_s \in \Lambda \subseteq M$ $(1 \leqslant s \leqslant k)$ such that $\sum_{\substack{1\leqslant s\leqslant k\\ n_s \mid x-a_s}} \lambda_s \in S$ for each $x \in \mathbb{Z}$. With the help of the generating theorem, we are able to generalize Znám's results on an inequality posed by Mycielski, and prove the following remarkable theorem:

Let P be a set of some primes, and f a mapping into an R-module K such that $\langle (x+r)/p, py\rangle \in Dom(f)$ for all $r = 0, 1, \cdots, p-1$ if $p \in P$ and $\langle x, y\rangle \in Dom(f)$. Then the following statements are equivalent:

(a) $\sum_{r=0}^{p-1} f(\frac{x+r}{p}, py) = f(x, y), \quad \forall p \in P, \langle x, y\rangle \in Dom(f)$.

(b) $\sum_{s=1}^{k} \lambda_s f(\frac{x+a_s}{n_s}, n_s y) = \sum_{t=1}^{l} \mu_t f(\frac{x+b_t}{m_t}, m_t y)$ *for all* $\langle x, y\rangle \in Dom(f)$ *whenever* $\sum_{\substack{1\leqslant s\leqslant k\\ n_s \mid x-a_s}} \lambda_s = \sum_{\substack{1\leqslant t\leqslant l\\ m_t \mid x-b_t}} \mu_t$ *for each* $x \in \mathbb{Z}$ $(\lambda_s, \mu_t \in R,\ 0 \leqslant a_s < n_s,\ 0 \leqslant b_t < m_t)$ *and any prime dividing* $\prod_{s,t} n_s m_t$ *is contained in* P.

In addition, some mappings satisfying (a) are listed in the paper.