SOME DIOPHANTINE REPRESENTATIONS RELATED TO $\binom{P \times}{Q \times}$

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ABSTRACT

Diophantine representations of binomial coefficients play important roles in the discussion about Hilbert’s tenth problem. In this paper, for fixed $P$ and $Q$, with only seven natural number unknowns we give Diophantine representations of the relations $Y = \binom{P \times}{Q \times}$ and $\binom{P \times}{Q \times} \equiv Y \pmod{Z}$. Furthermore, when $X > 1$, for any positive integer $N$ we may respectively require that the unknowns lie between $N$ and $\Phi(P, Q, X, Y, N)$, $N$ and $\Psi(P, Q, X, Y, Z, N)$, where $\Phi$ and $\Psi$ are Kalmar elementary functions. For the related relations $Z = C_X$ ($Z$ is the $X$-th Catalan number), $X$ pow 2 ($X$ is a power of 2) and the prime set (under Jones’ conjecture), we have similar results on their Diophantine representations.