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REPRESENTING n AS $n = x + y + z$ WITH $x^2 + y^2 + z^2$ A SQUARE

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Abstract. In this paper, we mainly confirm the following conjecture of Sun posed in 2013: Each positive integer n can be written as $n = x+y+z$ with x, y, z positive integers such that $x^2 + y^2 + z^2$ is a square, unless n has the form $n = 2^a 3^b$ or $2^a 7$ with a and b nonnegative integers.

1. Introduction

Lagrange's four-square theorem states that each $n \in \mathbb{N} = \{0, 1, 2, \ldots\}$ can be written as $x^2 + y^2 + z^2 + w^2$ with $x, y, z, w \in \mathbb{N}$. Z.-W. Sun [13, 14] refined this classical theorem in various ways by imposing certain restrictions involving squares. For example, Sun's 1-3-5 conjecture [13] states that any $n \in \mathbb{N}$ can be written as $x^2 + y^2 + z^2 + w^2$ $(x, y, z, w \in \mathbb{N})$ with $x + 3y + 5z$ a square. This was confirmed by Machiavelo and Tsopanidis [6] via Hamilton quaternions.

Squares are actually polygonal numbers of order four. In 1813 Cauchy proved Fermat's claim that for each integer $m \geq 5$, any $a \in \mathbb{N}$ can be written as a sum of m polygonal numbers of order m (i.e., those $p_m(n) = (m-2)\binom{n}{2} + n$ with $n \in \mathbb{N}$). The following lemma plays a central role in Cauchy's proof, which can be found in Nathanson [8] and [9, p. 31], or Moreno and Wagstaff [7, pp. 54-57].

Cauchy's Lemma. Let a and b be positive odd integers such that

$$b^2 < 4a \quad \text{and} \quad 3a < b^2 + 2b + 4. \tag{1.1}$$

Then there are $s, t, u, v \in \mathbb{N}$ such that

$$s + t + u + v = b \text{ and } s^2 + t^2 + u^2 + v^2 = a. \tag{1.2}$$

In view of Cauchy's Lemma, it is interesting to investigate partitions of integers with restrictions involving squares. Namely, given $k \in \{3, 4, 5, \ldots\}$, we investigate for which positive integers n the system of Diophantine equations

$$\begin{cases} x_1 + \cdots + x_k = n, \\ x_1^2 + \cdots + x_k^2 = y^2 \end{cases} \tag{1.3}$$

has solutions $x_1, \ldots, x_k, y \in \mathbb{Z}^+ = \{1, 2, 3, \ldots\}$.

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Our first theorem solves the above problem for $k \geq 4$.

Theorem 1.1. *Let $k \geq 4$ be an integer. Then any integer $n > \max\{20k, 1200\}$ can be written as $n = x_1 + \cdots + x_k$ with $x_1, \ldots, x_k \in \mathbb{Z}^+$ such that $x_1^2 + \cdots + x_k^2$ is a square.*

We will prove Theorem 1.1 in the next section using Cauchy's lemma. Actually, by our proof we may even taking $x_5 = \cdots = x_k = 2$ when $k \geq 5$ and $2 \nmid n$.

Our next theorem handles the case $k = 3$. It confirms a conjecture of Sun posed in 2013 (cf. [11] and [12, A230121]).

Theorem 1.2. *Let n be a positive integer. We can write $n = x + y + z$ with $x, y, z \in \mathbb{Z}^+$ such that $x^2 + y^2 + z^2$ is a square, if and only if n is neither of the form $2^a 3^b$ $(a, b \in \mathbb{N})$ nor of the form $2^a 7$ $(a \in \mathbb{N})$.*

Remark 1.1. This theorem also can be interpreted as the existence of certain cuboid (i.e., rectangular box) with integer edges and body diagonal. It is still open whether there exists a so-called perfect cuboid that has integer edges, face diagonals and body diagonal (cf. Section D18 of [5, pp. 275–283]).

As an example, Dickson [4, p. 259] recorded that Regiomontanus proposed in the 15th century the problem of solving the pair of equations

$$x + y + z = 116, \quad x^2 + y^2 + z^2 = 4624 = 68^2.$$

It has a unique solution in positive integers, up to permutations, namely $\{x, y, z\} = \{32, 36, 48\}$.

In contrast with Theorem 1.2, Sun ([13, Conjecture 4.7] and [15, Conjectures 1.31-1.36]) conjectured for many triples of positive integers a, b, c (such as $(a, b, c) = (1, 3, 12)$) that any $n \in \mathbb{N}$ can be written as $x^2 + y^2 + z^2 + w^2$ $(x, y, z, w \in \mathbb{N})$ with $ax^2 + by^2 + cz^2$ a square, but no progress has been made on this problem.

Our proof of Theorem 1.2 will be given in Section 4. To find suitable $x, y, z \in \mathbb{Z}^+$ in the "if" part, we need a lemma on representing certain positive integers as $x_0^2 + y_0^2 - 3z_0^2$ with $x_0 \geq z_0 > 0$ and $y_0 \geq 2z_0$, which will be provided in Section 3.

For convenience, we set $\Box = \{x^2 : x \in \mathbb{Z}\}$.

2. Proof of Theorem 1.1

Lemma 2.1. *Let m and n be positive odd integers with $n \geq 5$ and $3m < n^2 < 4m$. Then there are $s_0, t_0, u_0, v_0 \in \mathbb{Z}^+$ such that*

$$s_0 + t_0 + u_0 + v_0 = n \text{ and } s_0^2 + t_0^2 + u_0^2 + v_0^2 = m.$$

Proof. Let $a = m - 2n + 4$ and $b = n - 4$. Then it is easy to verify (1.1) holds. By Cauchy's Lemma, there are $s, t, u, v \in \mathbb{N}$ satisfying (1.2). Define

$$s_0 = s + 1,\ t_0 = t + 1,\ u_0 = u + 1,\ v_0 = v + 1.$$

Then

$$s_0 + t_0 + u_0 + v_0 = b + 4 = n$$

and

$$s_0^2 + t_0^2 + u_0^2 + v_0^2 = a + 2b + 4 = m.$$

This concludes the proof. □

Proof of Theorem 1.1. If $n \in \mathbb{N}$ has a desired representation, then so does $2n$. Thus it suffices to prove the theorem for any odd integer $n > \max\{10k, 600\}$.

Let $j = k - 4$ and consider the interval $I = (n/4 + 7j/2,\ n/3 + 10j/3)$. Suppose that I contains no odd square. Then, for some $h \in \mathbb{Z}$ we have

$$(2h-1)^2 \leq \frac{n}{4} + \frac{7j}{2} < \frac{n}{3} + \frac{10j}{3} \leq (2h+1)^2$$

and hence

$$4h = (2h+1)^2 - (2h-1)^2 \geq \frac{n}{12} - \frac{j}{6} > \frac{n}{15} > 40,$$

which implies $h > 10$. Thus

$$\frac{n}{4} + \frac{7j}{2} \geq (2h-1)^2 > 19(2h-1) > 36h > 9\left(\frac{n}{12} - \frac{j}{6}\right)$$

and hence $10j > n$, which contradicts our assumption.

By the above, there exists an odd integer m such that

$$\frac{n}{4} + \frac{7j}{2} < m^2 < \frac{n}{3} + \frac{10j}{3}, \tag{2.1}$$

and hence

$$3(m^2 - 4j) < n - 2j < 4(m^2 - 4j).$$

By Lemma 2.1, there are $x_1, x_2, x_3, x_4 \in \mathbb{Z}^+$ such that

$$x_1 + x_2 + x_3 + x_4 = n - 2j \quad \text{and} \quad x_1^2 + x_2^2 + x_3^2 + x_4^2 = m^2 - 4j.$$

Set $x_i = 2$ for $4 < i \leqslant k$. Then $\sum_{i=1}^k x_i = n$ and

$$\sum_{i=1}^k x_i^2 = m^2 - 4j + 2^2 j = m^2.$$

In view of the above, we have completed the proof of Theorem 1.1. □

3. A lemma on $x^2 + y^2 - 3z^2$

In this section, we study the representations by the indefinite anisotropic form $x^2 + y^2 - 3z^2$. Note that $x^2 + y^2 - 3z^2$ has small coefficients among integral ternary anisotropic forms. In [2, pp. 307–309], it is used to illustrate how to compute the group of integral automorphs for such forms, and its group is shown to possess mainly two generators:

$$\phi:\ (x, y, z) \mapsto (x, 3z - 2y, 2z - y) \text{ and } \psi:\ (x, y, z) \mapsto (3z - 2x, y, 2z - x).$$

Using this fact, we obtain the following lemma.

Lemma 3.1. *Let n be a positive integer with $n, n/6, n/7 \not\in \square$. Suppose that the equation*

$$n = x^2 + y^2 - 3z^2 \ (x, y, z \in \mathbb{Z}) \tag{3.1}$$

has solutions. Then, there are $x_0, y_0, z_0 \in \mathbb{Z}^+$ with $x_0^2 + y_0^2 - 3z_0^2 = n$ satisfying

$$x_0 \geq z_0 \quad \text{and} \quad y_0 \geq 2z_0. \tag{3.2}$$

Furthermore, we may require $x_0 > z_0$ if $n = x^2 - 2z^2$ for some $x, z \in \mathbb{Z}^+$ with $x/z \in (2, 7/2] \cup (5, +\infty)$.

Proof. If $n = x^2 + y^2$ with $x, y \in \mathbb{N}$, then we may assume $x \geq y > 0$ since $n \not\in \square$. Thus $n = x^2 + (2y)^2 - 3y^2$ and $(x_0, y_0, z_0) = (x, 2y, y)$ satisfies (3.2).

Now assume that n is not a sum of two squares. Choose a particular solution (r, s, t) of (3.1) with $r, s \in \mathbb{N}$ and

$$t = \min\{z \in \mathbb{Z}^+ : \ n = x^2 + y^2 - 3z^2 \text{ for some } x, y \in \mathbb{Z}\}.$$

In view of the integral automorphs ϕ and ψ, the equation (3.1) has three other solutions:

$$\phi(r, s, t) = (r, 3t - 2s, 2t - s), \tag{3.3}$$

$$\psi(r, s, t) = (3t - 2r, s, 2t - r), \tag{3.4}$$

$$\phi\psi(r, s, t) = (3t - 2r, 6t - 3r - 2s, 4t - 2r - s). \tag{3.5}$$

By the definition of t, we get $|2t - s| \geq t$ from the solution (3.3). So we have either $s \leq t$ or $s \geq 3t$. Similarly, by the solution (3.4), either $r \leq t$ or $r \geq 3t$. Since $r^2 + s^2 - 3t^2 = n$, one of r and s is greater than t and hence at least $3t$. If $r \geq 3t$ and $s \geq 3t$, then $(x_0, y_0, z_0) = (r, s, t)$ satisfies (3.2).

Now we handle the case $r \leq t$ and $s \geq 3t$. (The case $s \leq t$ and $r \geq 3t$ can be handled similarly.)

Suppose $s < 5t - 2r$. Then

$$-t < 4t - 2r - s \leq t - 2r \leq t.$$

By the definition of t and the solution (3.5), we must have $|4t - 2r - s| = t$ and hence $4t - 2r - s = t - 2r = t$. So $r = 0$ and $s = 3t$. It follows that $n = r^2 + s^2 - 3t^2 = 6t^2$, which contradicts $n/6 \not\in \square$.

By the last paragraph, we must have $s \geq 5t - 2r$. Note that the solution

$$(x_0, y_0, z_0) = \psi(r, s, t) = (3t - 2r, s, 2t - r)$$

satisfies (3.2) since

$$s \geq 2(2t - r), \quad 3t - 2r \geq 2t - r \text{ and } \ 2t - r \geq t > 0.$$

In view of the above, we have proved the first assertion of Lemma 3.1.

Now we prove the second assertion. Suppose that $n = x^2 - 2z^2$ for some $x, z \in \mathbb{Z}^+$ with $x/z \in (2, 7/2] \cup (5, +\infty)$. As $n/7 \not\in \square$, we have $x/z \neq 3$. We want to find a solution (x_0, y_0, z_0) of (3.1) satisfying (3.2) and the inequality $x_0 > z_0$.

Case 1. $x/z \in (2, 3)$, i.e., $0 < 2z < x < 3z$.

In this case, $(x_0, y_0, z_0) = (z, 2x - 3z, x - 2z)$ meets our purpose since

$$x^2 - 2z^2 = z^2 + (2x - 3z)^2 - 3(x - 2z)^2,$$
$$x_0 - z_0 = z - (x - 2z) = 3z - x > 0,$$
$$y_0 - 2z_0 = (2x - 3z) - 2(x - 2z) = z > 0.$$

Case 2. $x/z \in (3, 7/2)$, i.e., $0 < 3z < x \le 7/2z$.

Using the identity

$$n = x^2 - 2z^2 = (3x - 8z)^2 + (2x - 3z)^2 - 3(2x - 5z)^2,$$

we find that $(x_0, y_0, z_0) = (3x - 8z, 2x - 3z, 2x - 5z)$ meets our purpose as

$$x_0 - z_0 = (3x - 8z) - (2x - 5z) = x - 3z > 0,$$
$$y_0 - 2z_0 = (2x - 3z) - 2(2x - 5z) = 7z - 2x \ge 0.$$

Case 3. $x/z \in (5, 6)$, i.e., $5z < x < 6z$.

In this case,

$$n = x^2 - 2z^2 = (2x - 9z)^2 + (5z)^2 - 3(6z - x)^2$$

and hence $(x_0, y_0, z_0) = (2x - 9z, 5z, 6z - x)$ meets our purpose.

Case 4. $x/z \in [6, +\infty)$, i.e., $x \ge 6z$. In this case,

$$n = x^2 - 2z^2 = (5z)^2 + x^2 - 3(3z)^2$$

and hence $(x_0, y_0, z_0) = (5z, x, 3z)$ meets our purpose.

In view of the above, we have completed the proof of Lemma 3.1. □

4. Proof of Theorem 1.2

We need the following known result in the case $F(x, y, z) = x^2 + y^2 - 3z^2$.

Lemma 4.1. ([3, p. 164]) *Let p be an odd prime with $p \not\equiv 1 \pmod{24}$. Let $F(x, y, z)$ be any indefinite, anisotropic ternary quadratic form with integral coefficient matrix and determinant $-p$. Then*

$$\begin{aligned}&\mathbb{Z} \setminus \{F(x, y, z) : \ x, y, z \in \mathbb{Z}\}\\ =&\{4^k(8\ell + p) : \ k \in \mathbb{N}, \ \ell \in \mathbb{Z}\}\\ &\bigcup \left\{p^{2k+1}(p\ell + r^2) : \ k \in \mathbb{N}, \ \ell \in \mathbb{Z}, \ 1 \leqslant r \leqslant \frac{p-1}{2}\right\}.\end{aligned}$$

Remark 4.1. The reader may consult [10] for a more general result.

To prove Theorem 1.2, we also utilize the following parametrization of the solutions to the Diophantine equation $w^2 = x^2 + y^2 + z^2$ (which can be easily proved using Gaussian integers [1, pp. 161–162]):

$$\begin{aligned}w =&\frac{a^2 + b^2 + c^2 + d^2}{2}t, \quad x = \frac{a^2 + b^2 - c^2 - d^2}{2}t,\\ y =&(ac - bd)t, \quad z = (ad + bc)t.\end{aligned}$$

Proof of Theorem 1.2. (i) We first prove the "if" direction. If $m = x+y+z$ for some $x, y, z \in \mathbb{Z}^+$ with $x^2 + y^2 + z^2 \in \square$, then for any $q \in \mathbb{Z}^+$ we have $mq = qx+qy+qz$ with $(qx)^2+(qy)^2+(qz)^2 = q^2(x^2+y^2+z^2) \in \square$. Thus any $n \in \mathbb{Z}^+$ divisible by 21 or 49 has a desired representation since $21 = 2+5+14$ with $2^2 + 5^2 + 14^2 = 15^2$, and $49 = 1 + 18 + 30$ with $1^2 + 18^2 + 30^2 = 35^2$. If $n \in \mathbb{Z}^+$ does not belong to $\{2^a 3^b : a, b \in \mathbb{N}\} \cup \{2^a 7 : a \in \mathbb{N}\}$, then n is divisible by 21 or 49, unless it has a prime divisor $p \neq 2, 3, 7$.

It suffices to assume that n is an odd prime other than $3, 7$. We want to prove that there are $x, y, z \in \mathbb{Z}^+$ with

$$x + y + z = n \text{ and } x^2 + y^2 + z^2 \in \square.$$

(The assumption $n \neq 7$ will be useful in (4.3) below.)

Now, if a, b, c, d are integers with

$$2n = (a + c + d)^2 + (b + c - d)^2 - 3c^2 - 3d^2, \tag{4.1}$$

then, for

$$x = \frac{a^2 + b^2 - c^2 - d^2}{2},\ y = ac - bd,\ z = ad + bc, \tag{4.2}$$

we can verify $x + y + z = n$ and

$$x^2 + y^2 + z^2 = x^2 + (a^2 + b^2)(c^2 + d^2) = \left(\frac{a^2 + b^2 + c^2 + d^2}{2}\right)^2.$$

So, it suffices to find $a, b, c, d \in \mathbb{Z}$ satisfying (4.1) such that x, y, z given by (4.2) are positive.

As $2n$ is neither of the form $3^{2u+1}(3v + 1)$ $(u, v \in \mathbb{N})$ nor of the form $4^u(8v + 3)$ $(u, v \in \mathbb{N})$, in view of Lemma 4.1 we have $2n \in \{x^2 + y^2 - 3z^2 :\ x, y, z \in \mathbb{Z}\}$. By Lemma 3.1 there are integers $x_0, y_0, z_0 \in \mathbb{Z}^+$ with $2n = x_0^2+y_0^2-3z_0^2$ for which $x_0 \geq 2z_0$ and $y_0 \geq z_0$; moreover, we may require $y_0 > z_0$ if $2n = r^2 - 2s^2$ for some $r, s \in \mathbb{Z}^+$ with $r/s \in (2, 7/2] \cup [5, +\infty)$.

Case 1. $y_0 > z_0$.

In this case, we set

$$a = x_0 - z_0,\ b = y_0 - z_0,\ c = z_0,\ d = 0.$$

It is easy to see that (4.1) holds and also $a \geq c > 0$ and $b > 0$ so that x, y, z given by (4.2) are positive.

Case 2. $y_0 = z_0$.

In this case, $2n = x_0^2+y_0^2-3z_0^2 = x_0^2-2z_0^2$ and $x_0/z_0 \notin (2, 7/2] \cup (5, +\infty)$. Clearly, $x_0/z_0 \in \{2, 5\}$ contradicts the assumption that n is a prime. Hence $x_0/z_0 \in (7/2, 5)$.

If $x_0/z_0 \in (4, 5)$, then it is easy to see that the integers

$$a = x_0 - 2z_0,\ b = 2z_0 \text{ and } c = d = z_0$$

meet our purpose.

Now we assume that $7/2 < x_0/z_0 \leqslant 4$. If $x_0 = 4z_0$, then

$$2n = x_0^2 - 2z_0^2 = 14z_0^2, \tag{4.3}$$

which contradicts $n \neq 7$. Thus $7z_0/2 < x_0 < 4z_0$. Set

$$a = x_0 - 2z_0,\ b = 5z_0 - x_0,\ c = a,\ d = z_0.$$

Then

$$a + c + d = 2x_0 - 3z_0,\ b + c - d = 2z_0,\ c = x_0 - 2z_0,$$

and hence (4.1) holds. It is easy to see that $x > 0$ and $z > 0$. Note also that

$$\begin{aligned} y =& ac - bd = (x_0 - 2z_0)^2 - (5z_0 - x_0)z_0 \\ =& {x_0}^2 - 3x_0z_0 - z_0^2 = \left(x_0 - \frac{3}{2}z_0\right)^2 - \frac{13}{4}z_0^2 \\ >& 4z_0^2 - \frac{13}{4}z_0^2 > 0. \end{aligned}$$

This concludes our proof of the "if" direction.

(ii) Now we prove the "only if" direction. If n is even and x, y, z are positive integers with $x + y + z = n$ and $x^2 + y^2 + z^2 \in \square$, then $x^2 + y^2 + z^2$ is a multiple of 4 and hence none of x, y, z is odd. Thus $n/2 = x_0 + y_0 + z_0$ with $x_0^2 + y_0^2 + z_0^2 \in \square$, where $x_0 = x/2,\ y_0 = y/2,\ z_0 = z/2$ are positive integers. So it remains to prove that any $n \in \{7\} \cup \{3^b :\ b \in \mathbb{N}\}$ cannot be written as $x + y + z$ with $x, y, z \in \mathbb{Z}^+$ and $x^2 + y^2 + z^2 \in \square$. It is easy to check that this holds for $n = 3, 7$.

Now assume $n = 3^b$ for some integer $b \geq 2$. Suppose that $n = x + y + z$ with $x, y, z \in \mathbb{Z}^+$ and $x^2 + y^2 + z^2 \in \square$. If we don't have $x \equiv y \equiv z \pmod 3$, then exactly one of x, y, z is divisible by 3 since $x + y + z \equiv 0 \pmod 3$, and hence $x^2 + y^2 + z^2 \equiv 2 \pmod 3$, which contradicts $x^2 + y^2 + z^2 \in \square$. Thus $x \equiv y \equiv z \equiv \delta \pmod 3$ for some $\delta \in \{0, 1, 2\}$. Write $x = 3x' + \delta$, $y = 3y' + \delta$ and $z = 3z' + \delta$ with $x', y', z' \in \mathbb{Z}$. Then $x' + y' + z' = n/3 - \delta \equiv -\delta \pmod 3$ and hence

$$x^2 + y^2 + z^2 \equiv 6(x' + y' + z')\delta + 3\delta^2 \equiv -6\delta^2 + 3\delta^2 = -3\delta^2 \pmod 9.$$

As $x^2 + y^2 + z^2$ is a square, we must have $\delta = 0$. Thus $n/3 = x' + y' + z'$ with $(x')^2 + (y')^2 + (z')^2 = (x^2 + y^2 + z^2)/9 \in \square$. Continuing this process, we finally get that 3 can be written as $x + y + z$ with $x, y, z \in \mathbb{Z}^+$ and $x^2 + y^2 + z^2 \in \square$, which is absurd. This contradiction concludes our proof of the "only if" direction.

In view of the above, we have completed the proof of Theorem 1.2. $\square$

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