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SOME IDENTITIES ON LINEAR RECURRENT SEQUENCES OF SECOND ORDER

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Abstract

Let $w_{n+2} = Aw_{n+1} - Bw_n$ $(B \neq 0)$ for $n = 0, \pm 1, \pm 2, \cdots$. We determine completely when we have

$$w_{pn+r} = \sum_{k=0}^{n} \binom{n}{k} t^{n-k} s^k w_{qk+r} \quad \text{for all } n \in \mathbb{N} = \{0, 1, 2, \cdots\}.$$

Let $u_0 = 0$, $u_1 = 1$, and $u_{n+2} = Au_{n+1} - Bu_n$ for $n = 0, \pm 1, \pm 2, \cdots$. For any $l, m \in \mathbb{N}$ and function $f : \mathbb{N} \to \{k \in \mathbb{Z} : w_k \neq 0\}$, we show the following symmetric identity

$$\sum_{k=0}^{l-1} \frac{B^{f(k)} u_{f(k+m)-f(k)}}{w_{f(k)} w_{f(k+m)}} = \sum_{k=0}^{m-1} \frac{B^{f(k)} u_{f(k+l)-f(k)}}{w_{f(k)} w_{f(k+l)}},$$

which is then applied to compute the series $\sum_{k=0}^{+\infty} B^{f(k)} u_{f(k+m)-f(k)}/(w_{f(k)} w_{f(k+m)})$. Our results are generalizations of some work in [1][2][3][7][8].

Keywords: Linear recurrent sequence, Lucas sequence, Exponential generating function.