Dear Dr. Sun,

I just got your letter, but I have not yet found your paper. You can reach me now at my Budapest address but you can also send a copy to Prof. R.L. Graham, Bell Laboratories, Murray Hill, New Jersey 07941 USA.

Denote by \( f(m) \) the number of solutions of

\[
1 = \frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n} \quad \text{with} \quad 1 < x_1 \leq x_2 \leq \cdots \leq x_n \quad \text{are integers}
\]

I never saw an asymptotic formula for \( f(m) \).

Is it true that \( \max_{1 < x_i < x_{i+1}} x_i - x_{i+1} = 3 \)? \( 2, 3, 6 \) are the only solutions with \( \max_{1 < x_i < x_{i+1}} x_i - x_{i+1} = 3 \), \( x_m \to \infty \) as \( m \to \infty \)?

Find an integer \( m \) for which if you color the proper divisors of \( m \) by 2 colors then \( m \) is always the monochromatic sum of (distinct) divisors. I do not know if such a number exists. Such numbers exist probably for \( k \) colors—\( k \geq 3 \) is the smallest such number \( m \) (if it exists) must be quite large.

"kind regards, au revoir"

E.P.
Dear Dr. Sam

Many thanks for your letter and interesting paper.

I hope you had a pleasant and useful visit to Italy.

Let $a_1 (mod m_1), a_2 (mod m_2), \ldots, a_r (mod m_r)$, $m_1 < m_2 < \ldots < m_r$

set $f(m_r)$, offered and offer 1000 dollars for a proof of existence, I offered and offer 1000 dollars for a proof of existence.

I think $m_r = 24$ is the smallest $k$ for which such a system can exist. It is not even known that $m_r - f(m_r)$ is finite. Can you find a lower bound for $f(m_r)$? Also a bound for $m_r$ as a function of $m_1$. You can write me to the above address.

Kind regards

Paul Erdős