A fourth order approximation of fractional derivatives with its applications

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Abstract

A fourth-order compact difference approximation is derived for the space fractional derivatives by using the weighted average of the shifted Grunwald formulae combining the compact technique. The properties of proposed fractional difference quotient operator are presented and proved. Then the new approximation formula is applied to solving the space fractional diffusion equations. By the energy method, the proposed compact difference scheme is proved to be unconditionally stable and convergent in $L_2$ norm for both 1D and 2D cases. Several numerical examples are given to confirm the theoretical results.